Dr. Nagendra Kumar
Scientist 'D', IIT Delhi
August 9, 2023...to present

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Qualifications

- 2022-23 — **Postdoctoral Scholar,** Stanford University, CA, USA.
- 2016-21 — **PhD,** Indian Institute of Technology Guwahati, India.
- 2013-15 — **M.Sc.,** Indian Institute of Technology Guwahati, India.
- 2010-13 — **B.Sc.,** Banaras Hindu University, Varanasi, India.

Research Interest

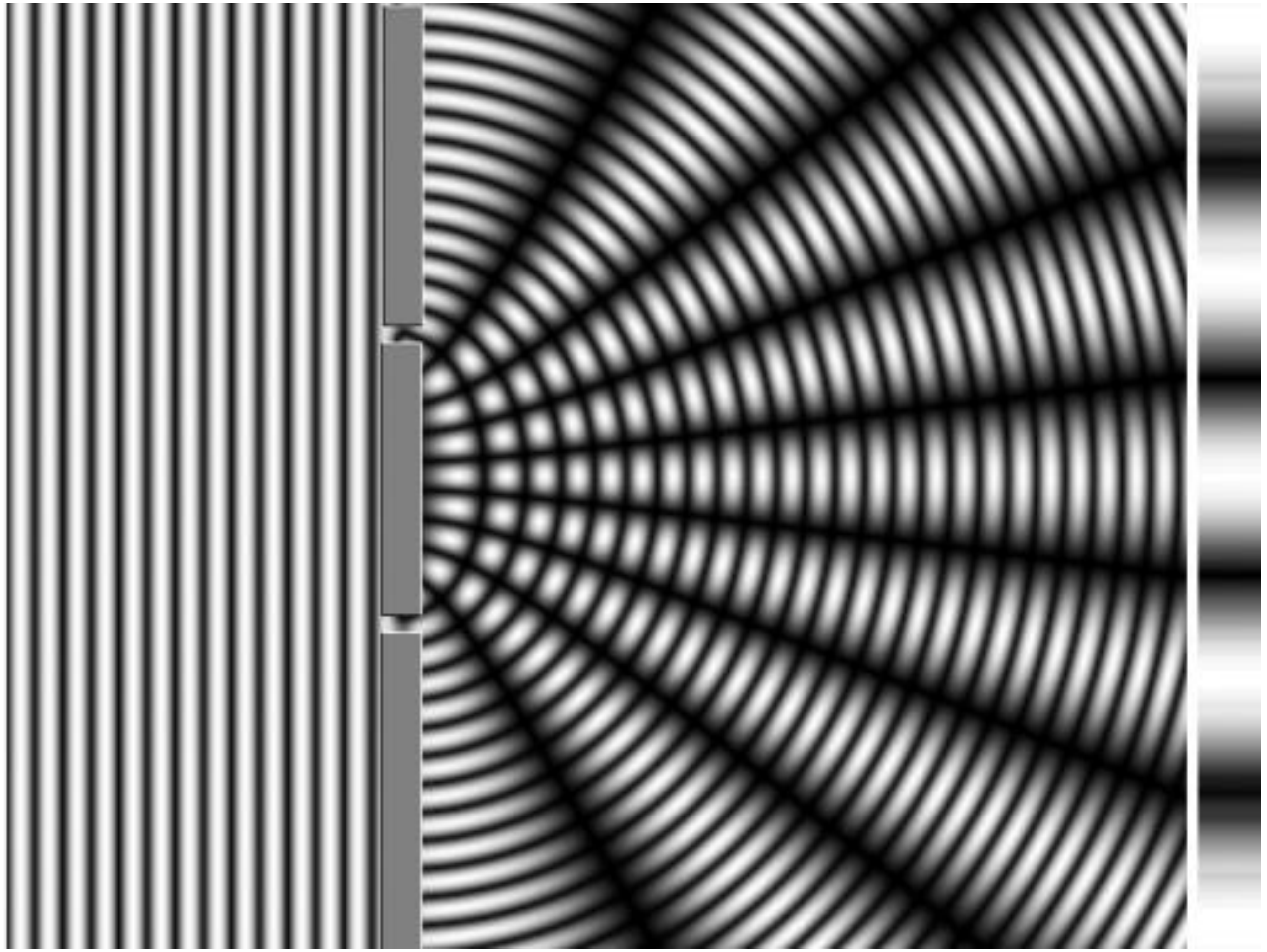
Wavefront sensing, diffractive optics, thin film deposition, Adaptive optics, retinal imaging, Quantum Optics (laser cooling and atom trapping and imaging).

Publications

12 Papers (peer-reviewed journals and proceedings), **1 Patent**, **6** Journal submitted**, 2** manuscript preparation, **4** book chapters.

Diffraction of Light

- Introduction
- Fraunhofer diffraction from a single slit
- Fraunhofer diffraction from a circular aperture
- Diffraction grating and its resolving power
- Fresnel's Half-Period Zones for Plane Wave.
- Fresnel diffraction pattern of a straight edge and at a circular aperture.
- Fraunhofer diffraction: Single slit, Double slit, Diffraction grating.
- Resolving power of grating.



Concept of Wavefront

- It is also defined as *a surface on which the wave disturbance is in same phase at all the points.*
- The direction of propagation of a wave at a point is always perpendicular to the wavefront through that point.
- Depending on source, the shape of the wavefront may be circular, spherical, cylindrical or planar.
- Point source produces spherical wavefront and linear source produces cylindrical wavefront.

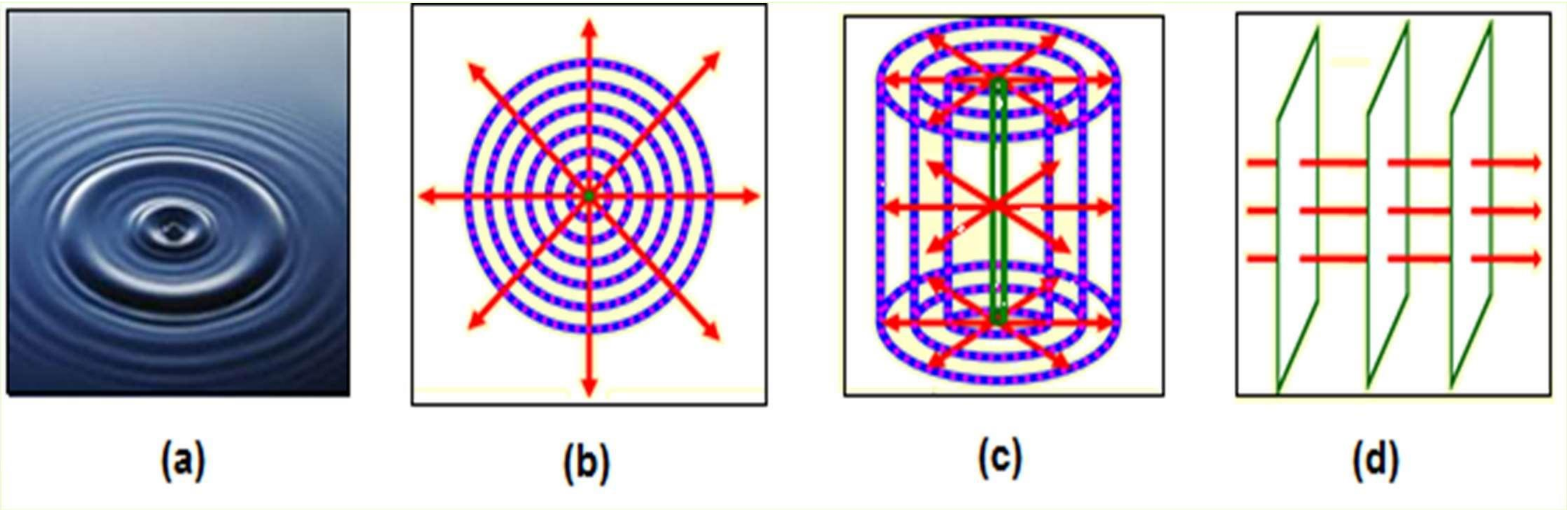
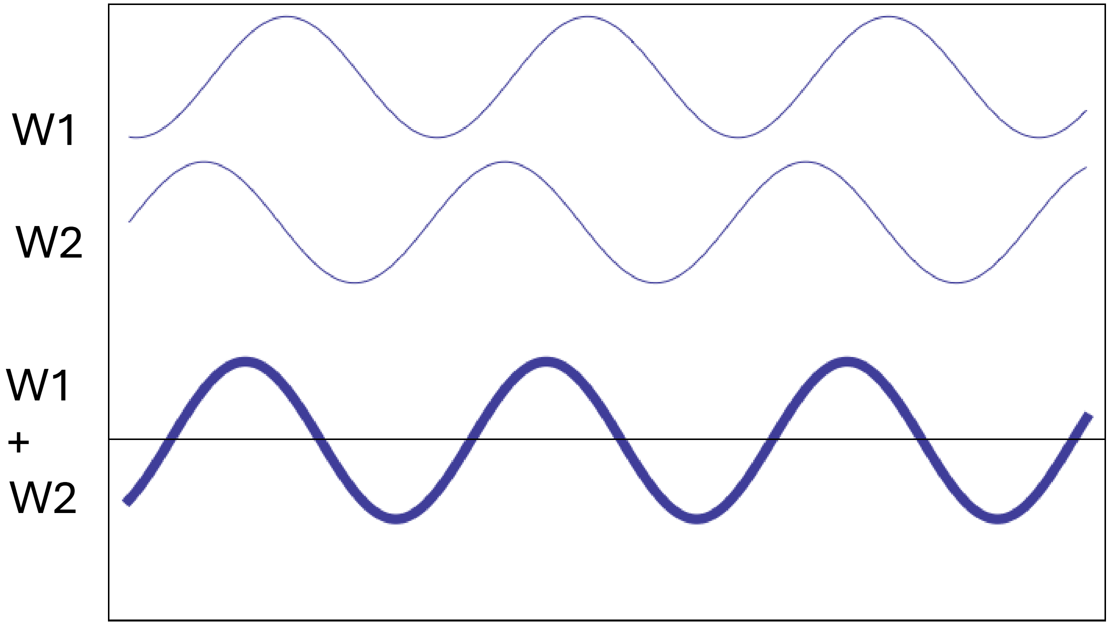
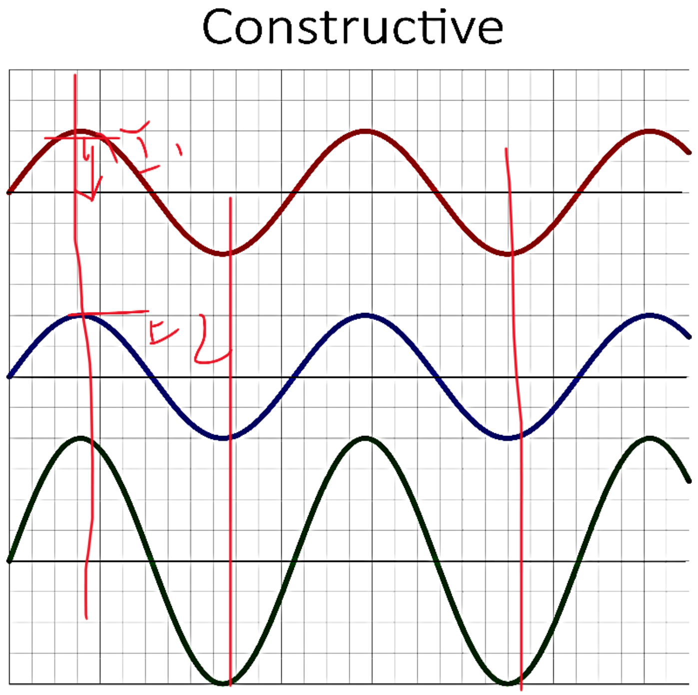


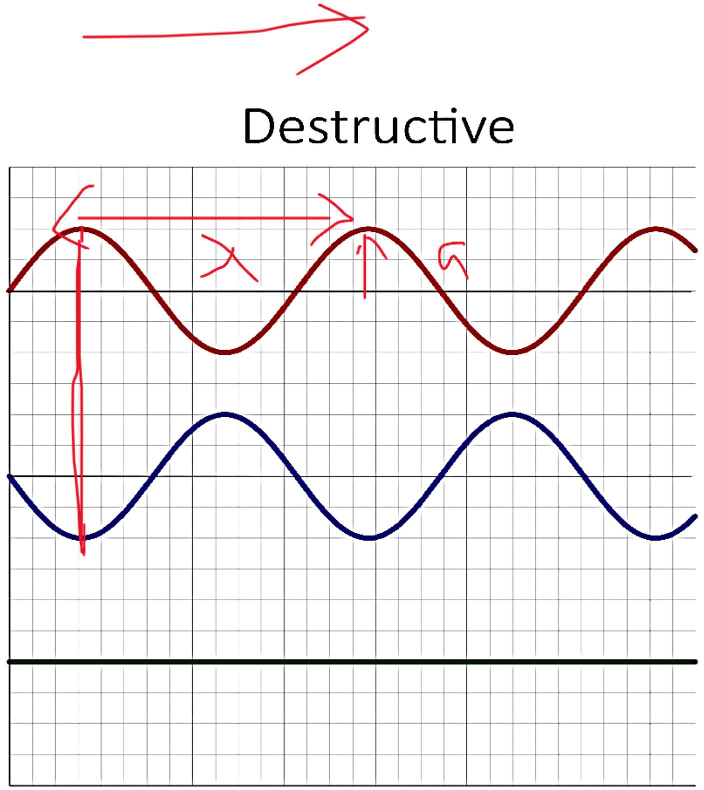
Figure1.(a) Circular (b) Spherical (c) Cylindrical (d) Plane Wavefront

- **Interference:** When *two or more coherent light waves traveling along the same direction superpose, then there is redistribution of light energy, at some points energy is maximum (constructive Interference) and at some points energy is minimum (destructive Interference). This phenomenon is called as Interference. This phenomenon is based on superposition principle.*

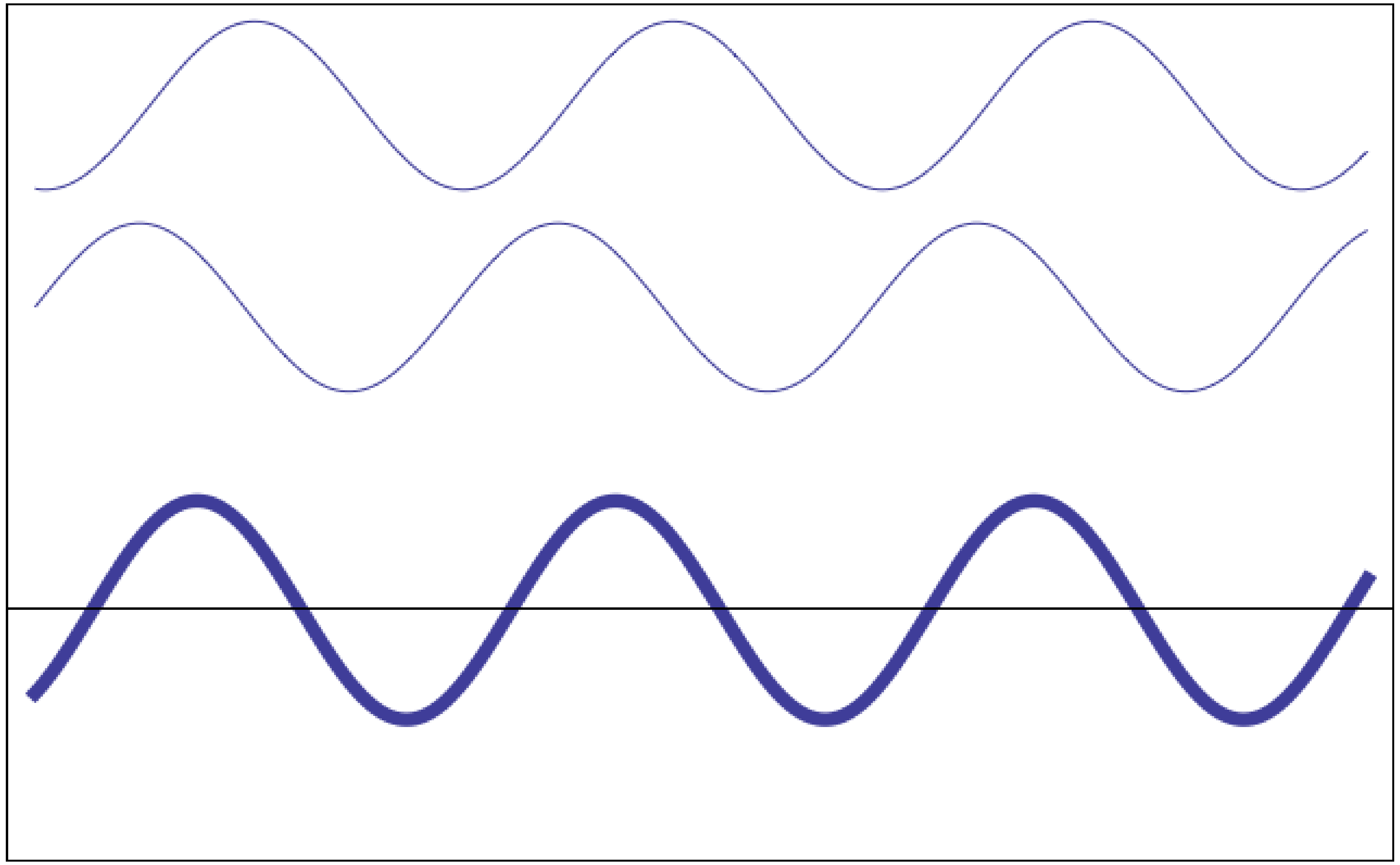
- ***Condition for constructive interference*:** When two light waves travelling through a medium arrive at a point in phase simultaneously, the resultant light intensity at that point is maximum and the point appears bright. This is called the condition of constructive interference.





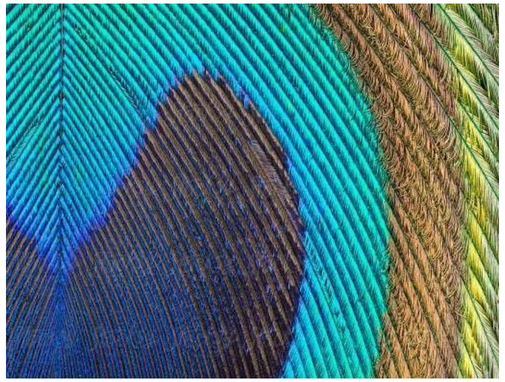


- The path difference between the two waves = nλ where n the order of fringe is 0, 1, 2, 3…..
- ***Condition for destructive interference*:** When two light waves travelling through a medium arrive at a point out of phase, the resultant light intensity at that point is minimum and the point appears dark. This is called the condition of destructive interference.
- The path difference between the two waves = (2n+1) the order of fringe is 1, 2, 3…..



Interfernce in thin films:Natural thin film and related phenomena: Iridescence Thin film:

A film is said to be thin when its thickness is of the order of one wavelength of visible light ~ 5500 Å (0.55μm) or of incident light.



Iridescence caused by interference : Colours as seen due to interference phenomena on oil layer, soap bubbles, peacock feathers, beetle body, oil film on rock and morph, butterfly wings.

Techniques of Interference

Division of wavefront

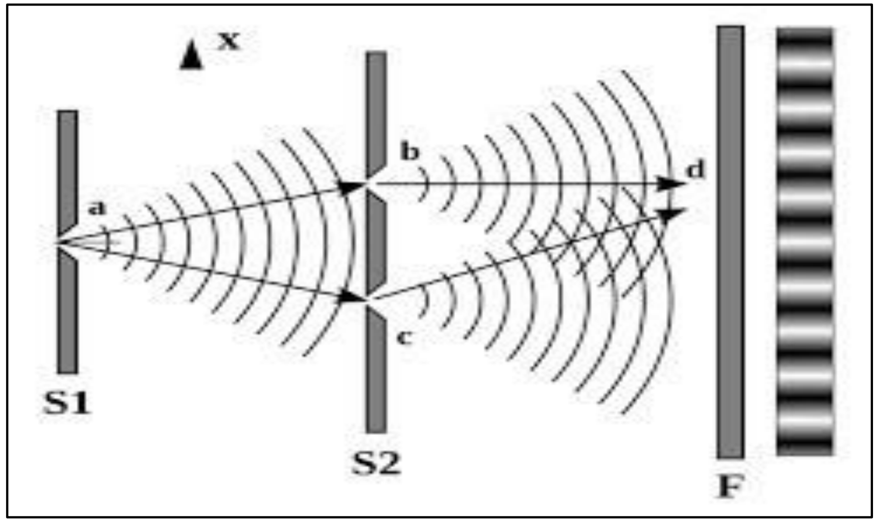


Figure 1

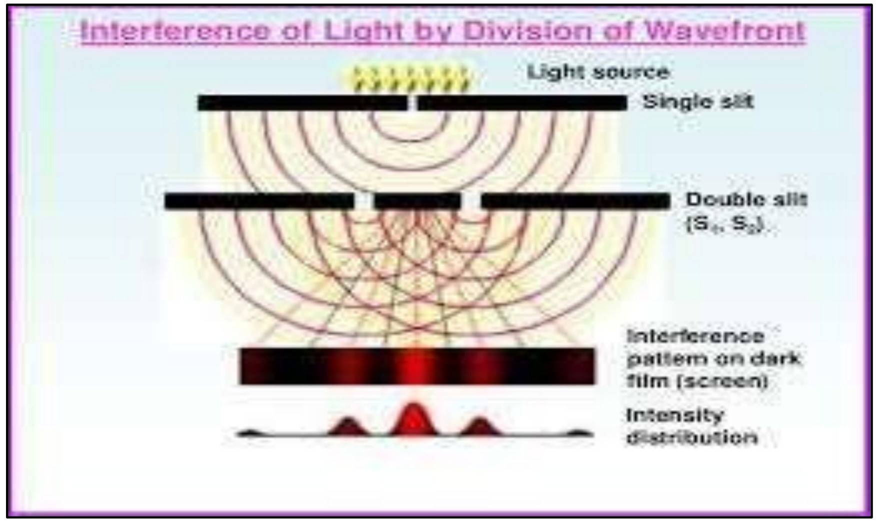


Figure 2

Division of amplitude

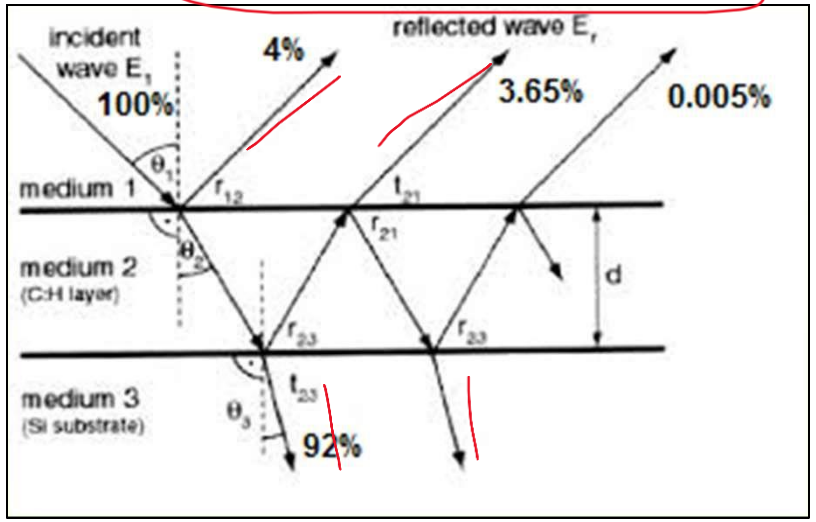


Figure 3

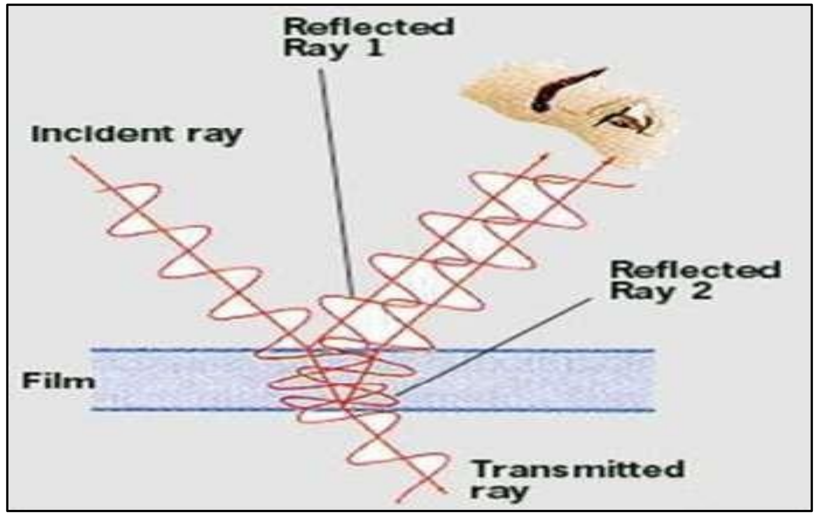
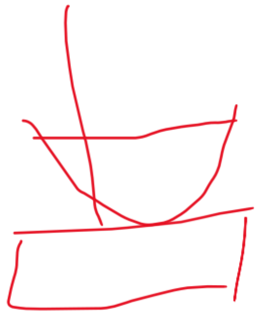


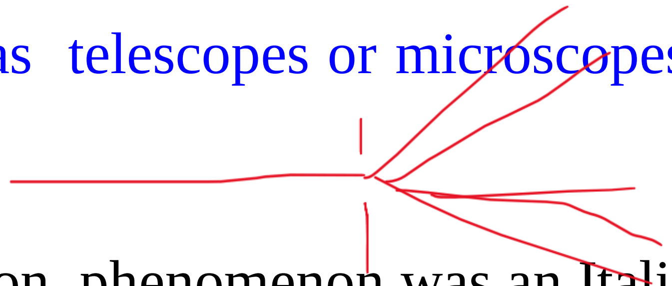
Figure 4

$I = A^2$



DIFFRACTION OF LIGHT: INTRODUCTION

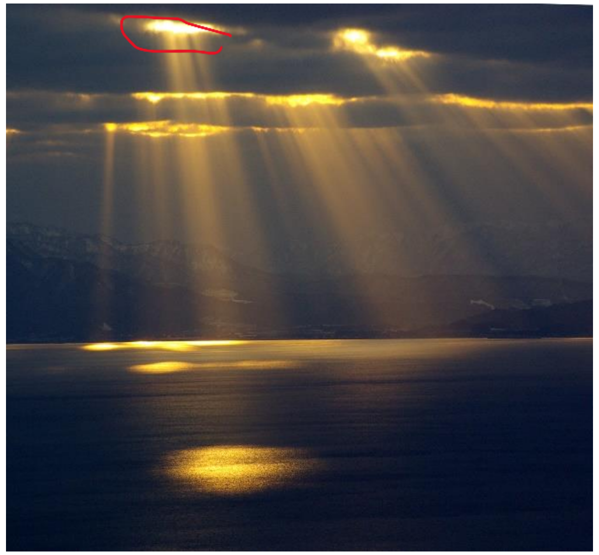
- Diffraction is exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves.
- Since the wavelength of light is much smaller than the dimensions of most of the obstacles; we do not encounter diffraction effects of light in everyday observations.
- Also, the finite resolution of our eye or of optical instruments such as telescopes or microscopes is limited due to the phenomenon of diffraction.



- The first scientist who recorded accurate observations on the diffraction phenomenon was an Italian scientist Francesco Maria Grimaldi, in 1660.
- He coined the word "diffraction" from the Latin word '*diffringere*', meaning 'to break into pieces', referring to light breaking up into different directions.
- Diffraction is defined as *~~the bending of light~~ rays around the corners of an obstacle or encroachment of light within the geometrical shadow of the obstacle or aperture.*

DIFFRACTION OF LIGHT:IN REAL LIFE

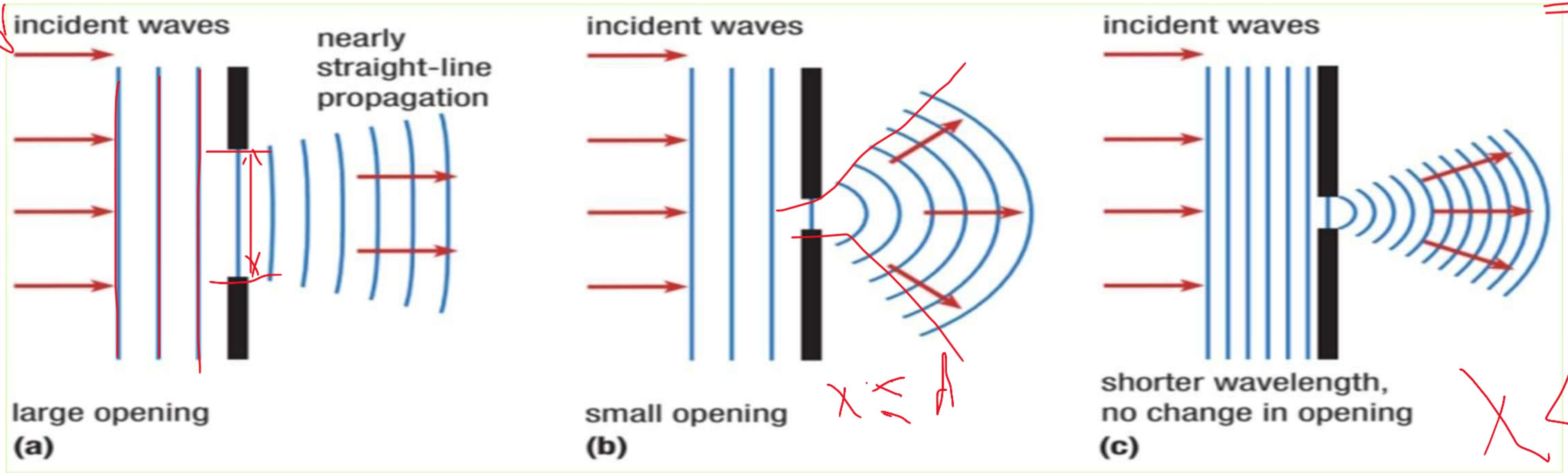
- Some examples of diffraction phenomenon in real life are formation of rainbow after rain, CD and DVD’s reflecting rainbow colours, Sun appears red during sunset, bending of light at the corners of the door as shown in figure
- Diffraction in the atmosphere by small particles can cause a bright ring to be visible around a bright light source like the sun or the moon.

Some examples of diffraction phenomenon in real life

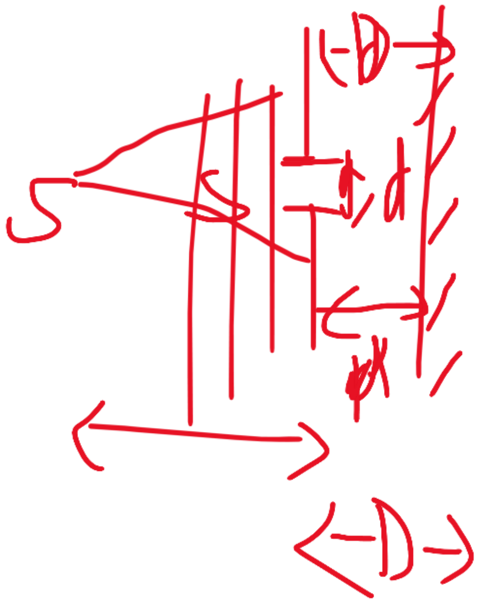
CONDITIONS FOR DIFFRACTION OF LIGHT

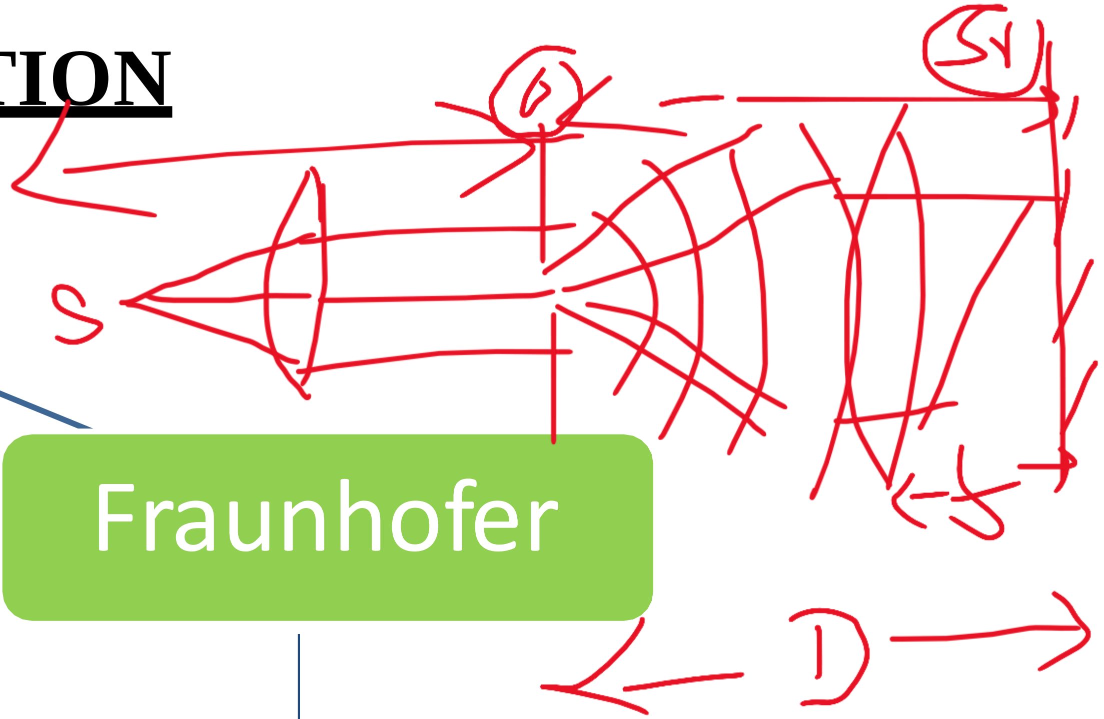
- The amount of diffraction depends on the wavelength of light and the size of the opening.
- If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable.
- *However, if the* size of the opening, *is of the order of* the wavelength of light, then *the amount of bending is considerable,i.e.the condition of diffraction.(Slit width should be of the the order of* the wavelength of light,
- An obstacle or opening will diffract shorter wavelength slightly and longer wavelengths more.



(a) and (b) As slit opening decreases, diffraction increases. (c) With shorter wavelength and no change in size of opening, diffraction decreases.

TYPES OF DIFFRACTION





Types of Diffraction

Fresnel

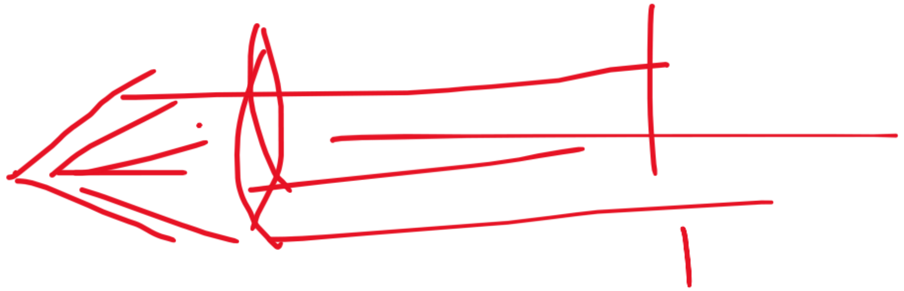
Fraunhofer

Fresnel

- The source or the screen or both are at finite distances from the obstacle (or aperture).
- In this case, no lenses are used to make the rays parallel or convergent.
- The incident wavefronts are either spherical or cylindrical.

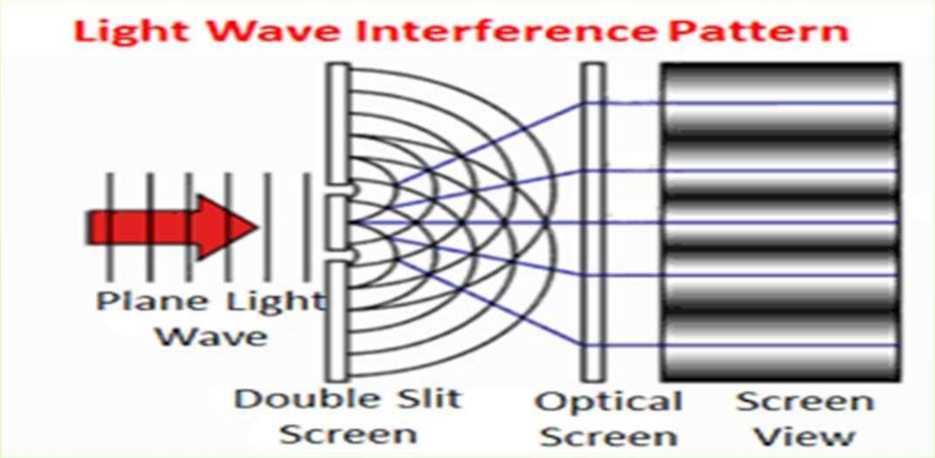
Fraunhofer

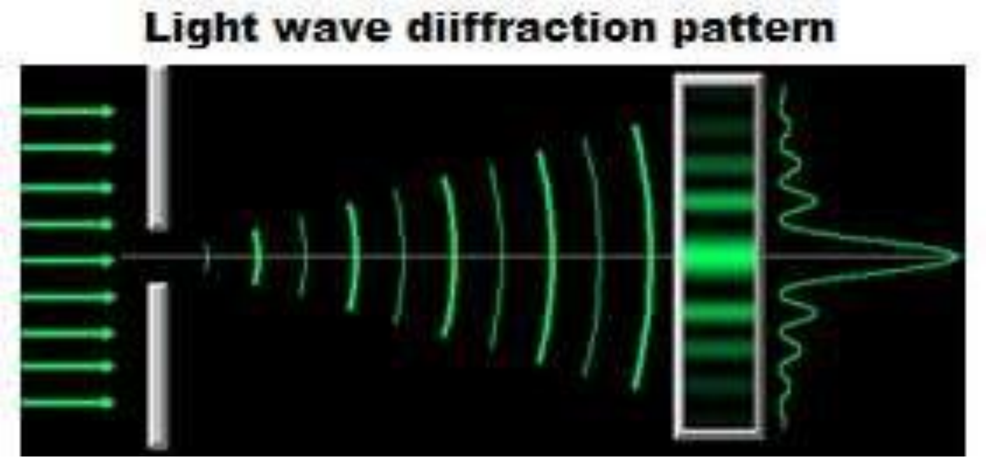
- The source and the screen or the telescope is placed at infinite distance from the obstacle.
- Lenses are used to make the rays converge.
- The incident wavefront on the aperture or obstacle and the telescope is plane wavefront.



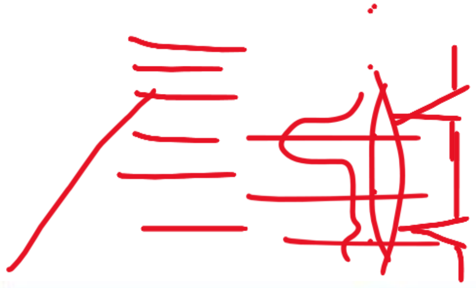
Difference between interference and diffraction

S.No.	Interference	Diffraction
1.	Interference phenomenon is due to superposition of light waves from two separated wavefronts.	Diffraction phenomenon is due to superposition of secondary wavelets originating from different points of the exposed parts of the same wavefront.
2.	In the interference pattern, the contrast between maxima and minima is good.	In the diffraction pattern, the contrast between maxima and minima is poor.
3.	In the interference pattern, regions of minimum intensity are perfectly dark and all bright fringes are of equal intensity.	In the diffraction pattern, regions of minimum intensity are not perfectly dark and only the first maxima has maximum intensity and the intensity decreases as the order of maxima increases.





FRAUNHOFER DIFFRACTION THROUGH SINGLE SLIT



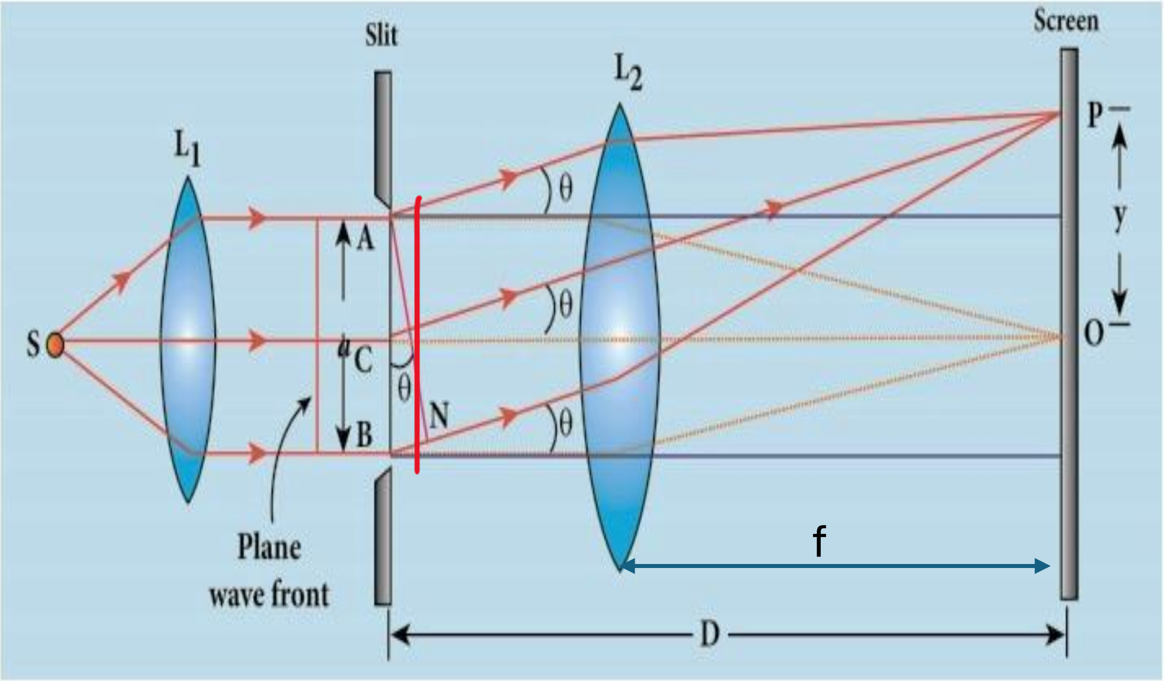
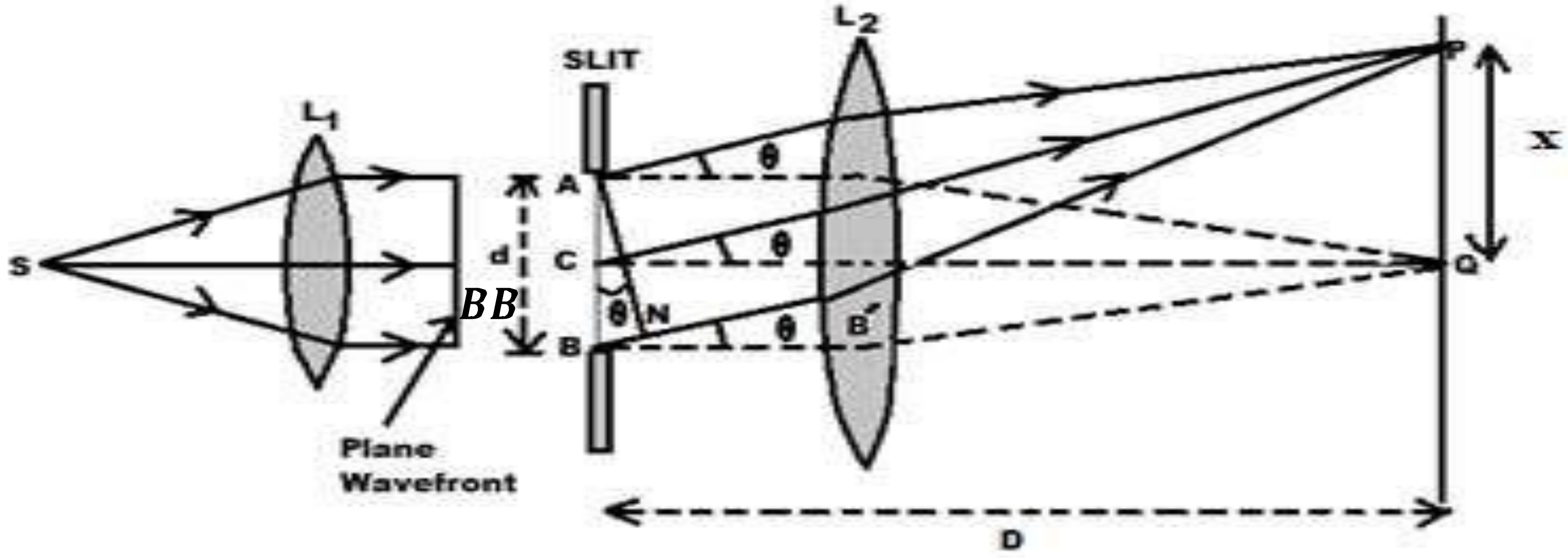
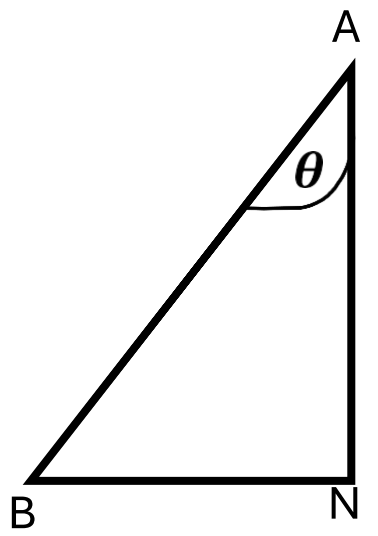


Fig: Fraunhofer diffraction through single slit

- Experimental arrangement is shown in figure
- The wavefront from source S is incident on the slit AB of width'd'.
- According to Huygens' Principle, each point of wavefront passing through the slit AB acts as a source of secondary wavelets.
- A real image of diffraction pattern is formed on the screen with the help of converginglensL_2.
- Thus, diffraction pattern on screen consists of a central bright band and alternate dark and bright bands of decreasing intensity on both sides.
- Let C be the center of the slit AB. The secondary waves, from points equidistant from center C of the slit lying on portion CA and CB of wave front travel the same distance in reaching Q and hence the path difference between them is zero.
- **These waves reinforce each other and give rise to the central maximum at point O.**

Derivation of width of slit and position of minima and secondary maxima.





The path difference between the secondary wavelets
Draw AN perpendicular on originating from A and B is BN.

From Δ BAN, $\frac{\boldsymbol{BN}}{\boldsymbol{AB}}$ = sinθ or BN = AB sinθ

Path difference, BN = d sin θ ≈ dθ (as θ is small),
For Minima: If the path difference is equal to one wavelength, i.e., BN= d sin θ = λ, position P will be of minimum intensity.

For first minima, for θ = $\boldsymbol{\theta_1}$, d sin $\boldsymbol{\theta_1}$ = λ or sin $\boldsymbol{\theta_1} = \frac{\lambda}{d}$

$\boldsymbol{\theta_1} = \frac{\lambda}{d}$ (for very small value of θ)

In general, for minima of order 'm', d sin$\boldsymbol{\theta_m}$= m λ

or sin$\boldsymbol{\theta_m} = \frac{m\lambda}{d}$

From the fig, ΔBAN,

Path difference $BN = AB\sin\theta$

$$\boxed{BN = d\sin\theta}$$

therefore, total phase difference, $= \frac{2\pi}{\lambda} \times$ path diff

$$= \frac{2\pi}{\lambda} . d\sin\theta \quad ----(1)$$

let us consider the width of the slit is divided into equal parts and the amplitude of the way from each parts is a. then the phase difference (p) is

$$p = \frac{1}{n} \times \text{total phase difference}$$

$$p = \frac{1}{n} \times \frac{2\pi}{\lambda} \times d\sin\theta$$

$$p = \frac{2\pi}{n\lambda} d\sin\theta \quad --- (2)$$

we know that resultant amplitude for n simple Harmonic motion,

$$R = \frac{a \sin(n\frac{p}{2})}{\sin(p/2)}. \quad --(3)$$

Now putting the value of p from eq^n (2), in (3).

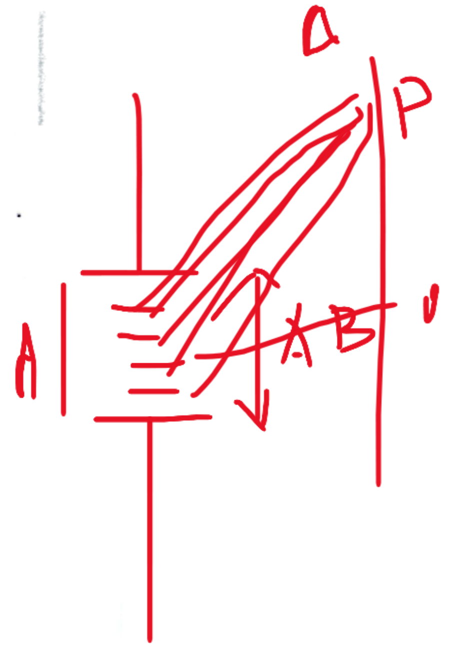
$$R = \frac{a \sin \frac{n}{2}\left(\frac{2\pi}{n\lambda}.d\sin\theta\right)}{\sin\frac{1}{2}\left(\frac{2\pi}{\lambda}.d\sin\theta\right)}$$

$$R = \frac{a \sin\left(\frac{\pi}{\lambda} d\sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda} d\sin\theta\right)}$$

let $\alpha = \frac{\pi}{\lambda} d\sin\theta$, then

$$R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

let α is very small, then $\frac{\alpha}{n}$ is very small. then $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$



$$R = \frac{a \sin\alpha}{(\alpha/n)} = n\alpha \frac{\sin\alpha}{\alpha}$$

if $A = n\alpha$, then $R = A\frac{\sin\alpha}{\alpha}$ ... (*)

$I \propto R^2$

$$\boxed{I = \frac{A^2 \sin^2\alpha}{\alpha^2}} \Rightarrow \boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2} \text{ — (4)}$$

Principal Maxima (central Maxima)

expand Eqn (*)

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \cdots\right]$$

since, $\left(\sin\alpha = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \cdots\right)$

$$R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right]$$

if negative terms minimizes, the R will be maximum.

i.e $\alpha = 0$

$$\alpha = \frac{\pi}{\lambda} d\sin\theta \Rightarrow 0 = \frac{\pi d\sin\theta}{\lambda}$$

we know that $\boxed{\sin 0^\circ = 0 = \sin\theta}$

therefore, central Maxima will be at $\boxed{\theta = 0^\circ}$ when maximum intensity is observed.

Minimum Intensity position :—

The intensity will be minimum, when $\sin\alpha = 0$

then, $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$

or $\alpha = \pm n\pi$ ....(5) when $n = 1, 2, 3 \cdots$

as $R = \frac{A\sin\alpha}{\alpha}$, because $A \neq 0$, $\alpha \neq 0$

but $\cancel{\alpha = \frac{\pi d\sin\theta}{\lambda}}$ $\alpha = \frac{\pi d\sin\theta}{\lambda}$ — (6)

from Eqn (5) & (6)

$$\therefore \pm n\cancel{\pi} = \frac{\cancel{\pi} d\sin\theta}{\lambda} \Rightarrow \boxed{d\sin\theta = n\lambda}$$

$$\boxed{d \sin\theta = \pm n\lambda} \quad \text{---} (7)$$

this is the condition for minimum intensity.

Secondar Maxima:–

In order to locate the positions of secondary maxima, we apply mathematical method for minima.

let us differential. Eqn

$$I = \left(\frac{A\sin\alpha}{\alpha}\right)^2$$

$$\frac{dI}{d\alpha} = \left(\frac{2A\sin\alpha}{\alpha}\right) \cdot \frac{d}{d\alpha}\left(\frac{A\sin\alpha}{\alpha}\right)$$

$$\frac{dI}{d\alpha} = \frac{2A^2\sin\alpha}{\alpha} \cdot \left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right]$$

~~0 = 2A sin α~~ for maxima and minima

$$\frac{dI}{d\alpha} = 0$$

$\therefore$

$$\frac{2A^2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

therefore,

$$\frac{2A^2\sin\alpha}{\alpha} = 0$$

$$\boxed{\sin\alpha = 0}$$

but this was the condition for central maxima, therefor for secondary maxima,

or $\alpha\cos\alpha - \sin\alpha = 0$

$\therefore$ $\alpha = \tan\alpha$

$$\boxed{\alpha = \tan\alpha}$$

$$\alpha = \tan\alpha$$

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} + \cdots \pm\frac{(2n+1)\pi}{2}$$

we know, $\alpha = \frac{\pi}{\lambda} d\sin\theta$

therefore,

$$\boxed{d\sin\theta = \frac{(2n+1)\lambda}{2}}$$

thus we pha' path difference is the mullible of $\lambda/2$ then there will be scondary maxima.

Intensity pattern of diffracted light through single slits

$$\boxed{I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2} \text{ —(1)}$$

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi\ldots}{2}\ldots..$$

Or in general,

Substituting these value of in the equation(1), we get

$$I_{0_{\alpha\to 0}} = \left(\frac{\mathrm{A0}\ \sin 0}{0}\right)^2 = A_0^{\ 2}$$

$$I_{0_{\alpha\to\frac{3\pi}{2}}} = \left(\frac{\mathrm{A0}\ \sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4}{9\,\pi^2} A_0^{\ 2} = \frac{A_0^{\ 2}}{22}$$

$$I_{0_{\alpha\to\frac{5\pi}{2}}} = \left(\frac{\mathrm{A0}\ \sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{4}{25\pi^2} A_0^{\ 2} = \frac{A_0^{\ 2}}{61}$$

Thus, the intensity of the successive maxima are in the ratio

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \ldots\ldots.$$

Cleary, most of the incident light is concentrated in the principal maxima which occurs in the direction given by $\alpha = 0$

For secondary maxima:

If path difference, BN = d sin θ is an odd multiple of $\frac{\lambda}{2}$,

$$\text{i.e., } d \sin\theta_m = \frac{(2m+1)\lambda}{2}$$

Since θ_m is very small, $\sin \theta_m = \theta_m$,

$$\therefore \ \theta_m = \frac{(2m+1)\lambda}{2d}$$

where m=1 ,2, 3, …. is an integer.

Width of central maximum: 2x

- Let 'f' be the focal length of lens L_2 .
- The distance of first minima on either side of the central maxima be $'x'$ as shown in fig. 23.

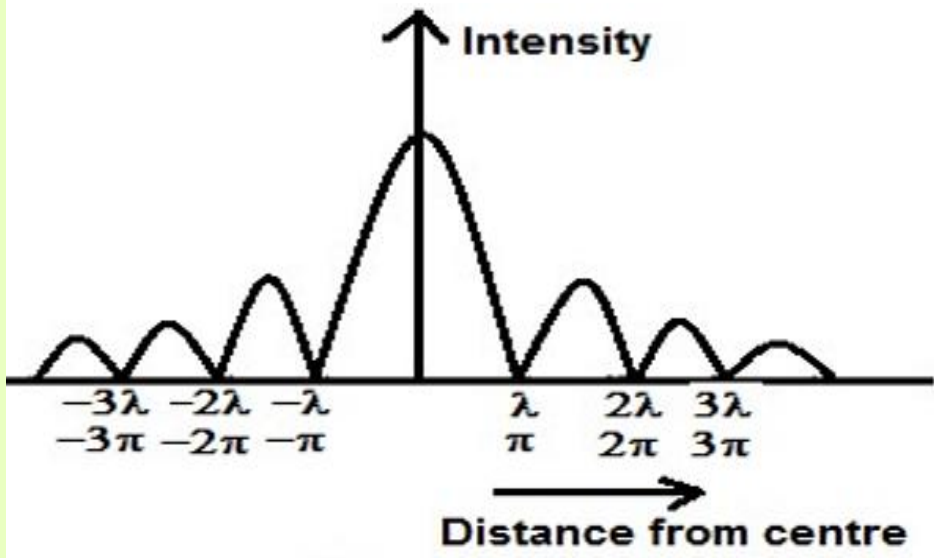
Then, $\tan\theta = \frac{x}{f}$

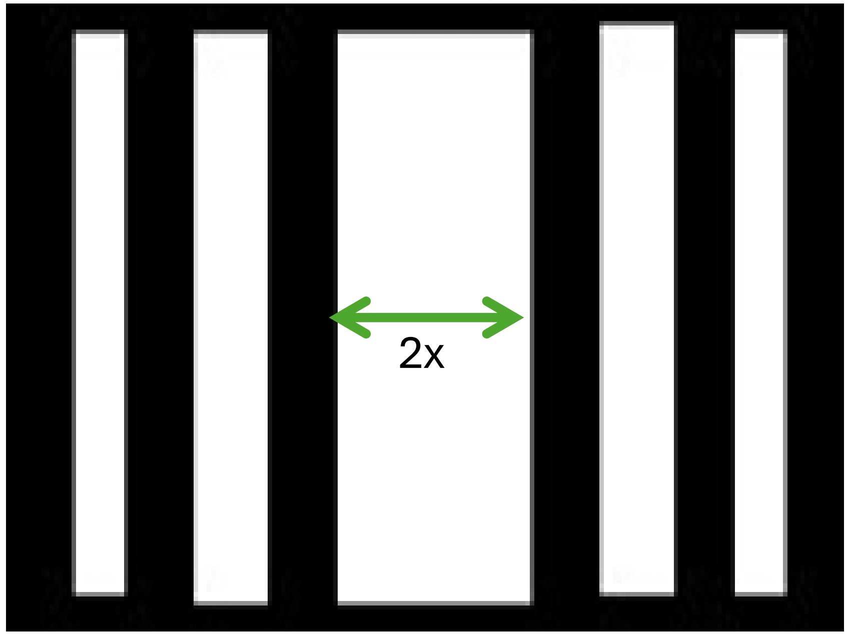
- Since the lens L_2 is very close to the slit, so f = D, $\tan\theta = \frac{x}{D}$ ---------- (1)
- For θ is very small, $\tan\theta \approx \sin\theta$, $\therefore$ $\tan\theta = \sin\theta = \frac{x}{D}$ ---------- (2)
- Also, for first minima, $d\sin\theta = \lambda$ or $\sin\theta = \frac{\lambda}{d}$ ---------- (3)
- From eqns. (2) and (3), we have $\frac{x}{D} = \frac{\lambda}{d}$

$$\text{or } x = \frac{\lambda}{d}D \qquad \text{---------- (4)}$$

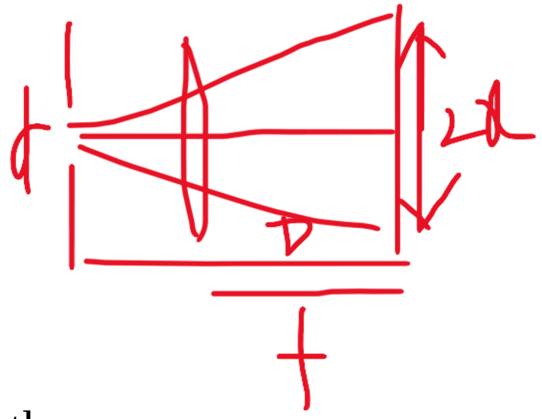
This is the distance of first minima on either side from the centre of the central maximum.

- Width of central maximum, $2x = \frac{2\lambda}{d}D$ ----------(5)





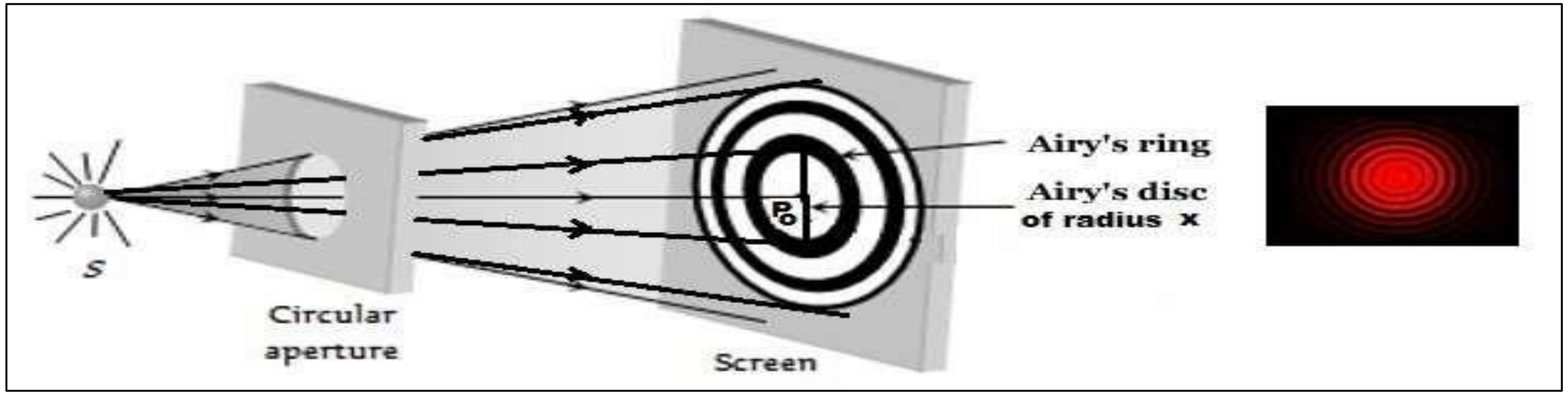
Width of central maximum, $2x = \frac{2\lambda}{d}D$



Observation:

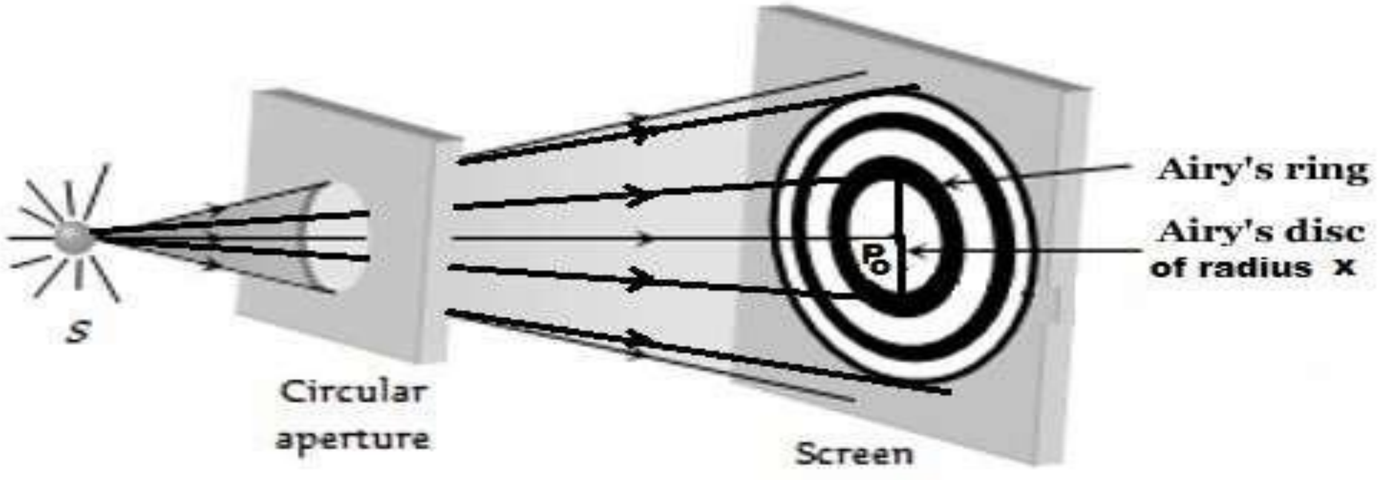
- The width of the central maximum is proportional to the wavelength of light.
- For longer wavelengths, the width of the central maxima is more than with light of shorter wavelength.
- With a narrow slit (smaller ‘d’ value), the width of the central maximum is more.
- The diffraction pattern consists of alternate bright and dark bands with monochromatic light.
- With white light, the central maximum is white and the rest of the diffraction bands are coloured.
- We can see that the maxima and minima are very close to the central maximum.
- But with a narrow slit ‘d’ is small and hence θ is large.(because $d \sin \theta = \lambda$)
- Hence, distinct diffraction maxima and minima are on both sides of the central maximum.

FRAUNHOFER DIFFRACTION THROUGH CIRCULAR APERTURE



- When a parallel beam of light is passed through circular aperture of an opaque board, then the light is diffracted by the aperture.
- If received on a screen at a large distance, the pattern is a bright disc called Airy disc surrounded by alternate dark and bright concentric rings called Airy rings of decreasing intensity.
- The wavefront is obstructed by the opaque board and only the points of the wavefront that are exposed by the aperture send the secondary wavelets.
- The bright and dark rings are formed by the superposition of these wavelets.
- The diffracted secondary wavelets are converged on the screen by keeping a convex lens between the aperture and the screen. The screen is at the focal plane of the convex lens.

FRAUNHOFER DIFFRACTION THROUGH CIRCULAR APERTURE



- **The secondary wavelets travel same distance to reach P_o and there is no path difference between these rays. Hence a bright spot is formed at P_o which is known *Airy's disc*. P_o corresponds to the central maximum.**

- **The secondary waves which travel at a certain angle θ with respect to central axis form a cone and hence, they form a diffracted ring on the screen.**
    - **The radius of Airy's disc is given by $x = 1.22\,\frac{\lambda f}{d}$**

- **Therefore, the radius of Airy's disc is inversely proportional to the diameter of the aperture. Therefore, by decreasing the diameter of aperture, the size of Airy's disc increases.**
- **Since the lenses used as objective and eyepieces in telescopes and microscopes are circular in shape and constitute a circular aperture. Hence Fraunhofer diffraction using circular Aperture is of the most practical interest.**

DIFFRACTION GRATING

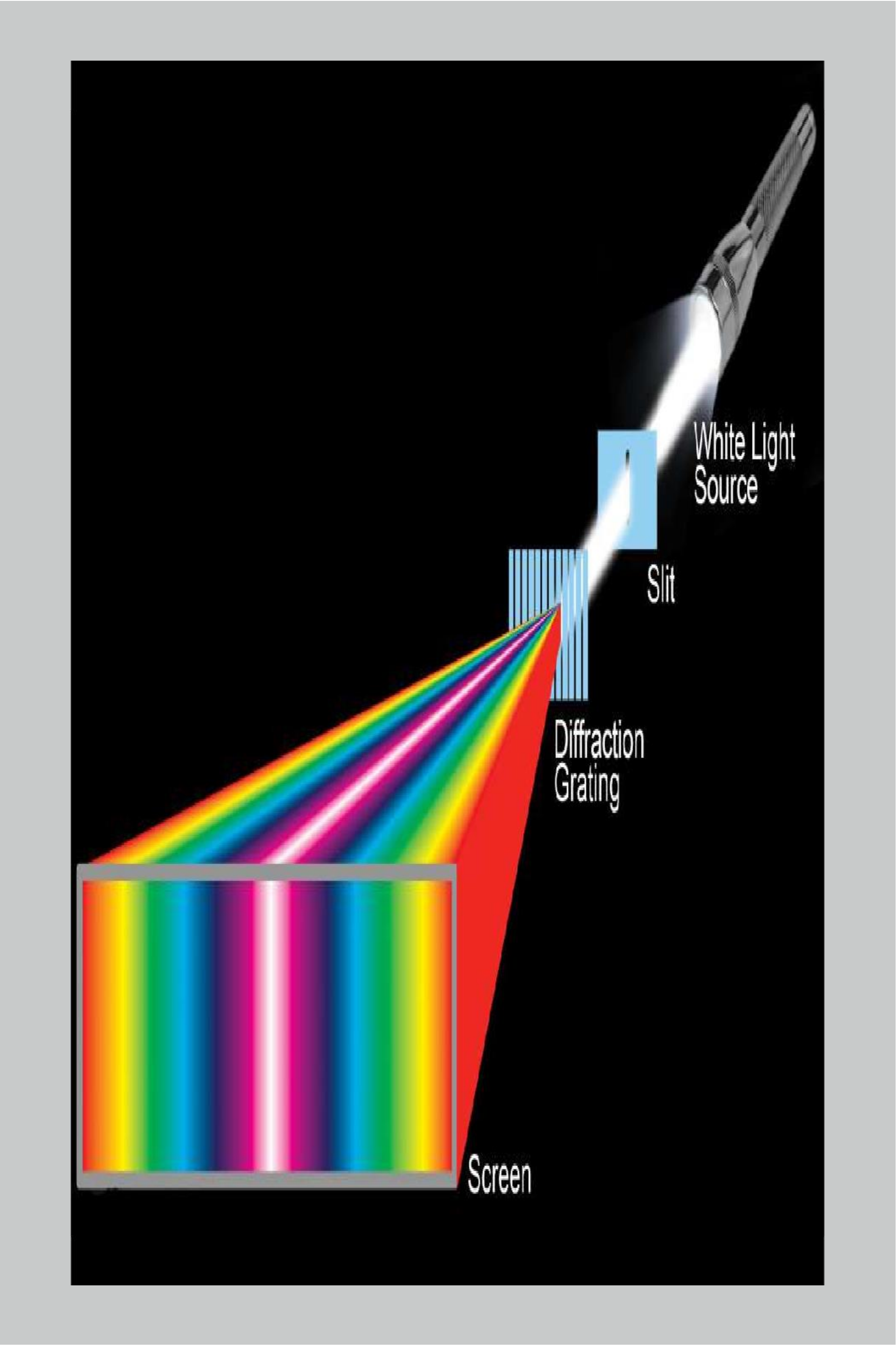
- Diffraction Gratings are optical components used to separate light into its component wavelengths.
- Diffraction Grating consists of a series of closely packed grooves(deep line cut in a surface) that have been engraved or etched into the Grating's surface.
- Plane transmission Grating is a plane sheet of transparent material on which opaque rulings are made with a fine diamond pointer.
- Thus, it consists of a large number of equally spaced parallel transparent spaces called slits.
- Grating element and grating equation:

If light is incident normally on a transmission grating of wavelength λ, then the direction of principal maxima is given by

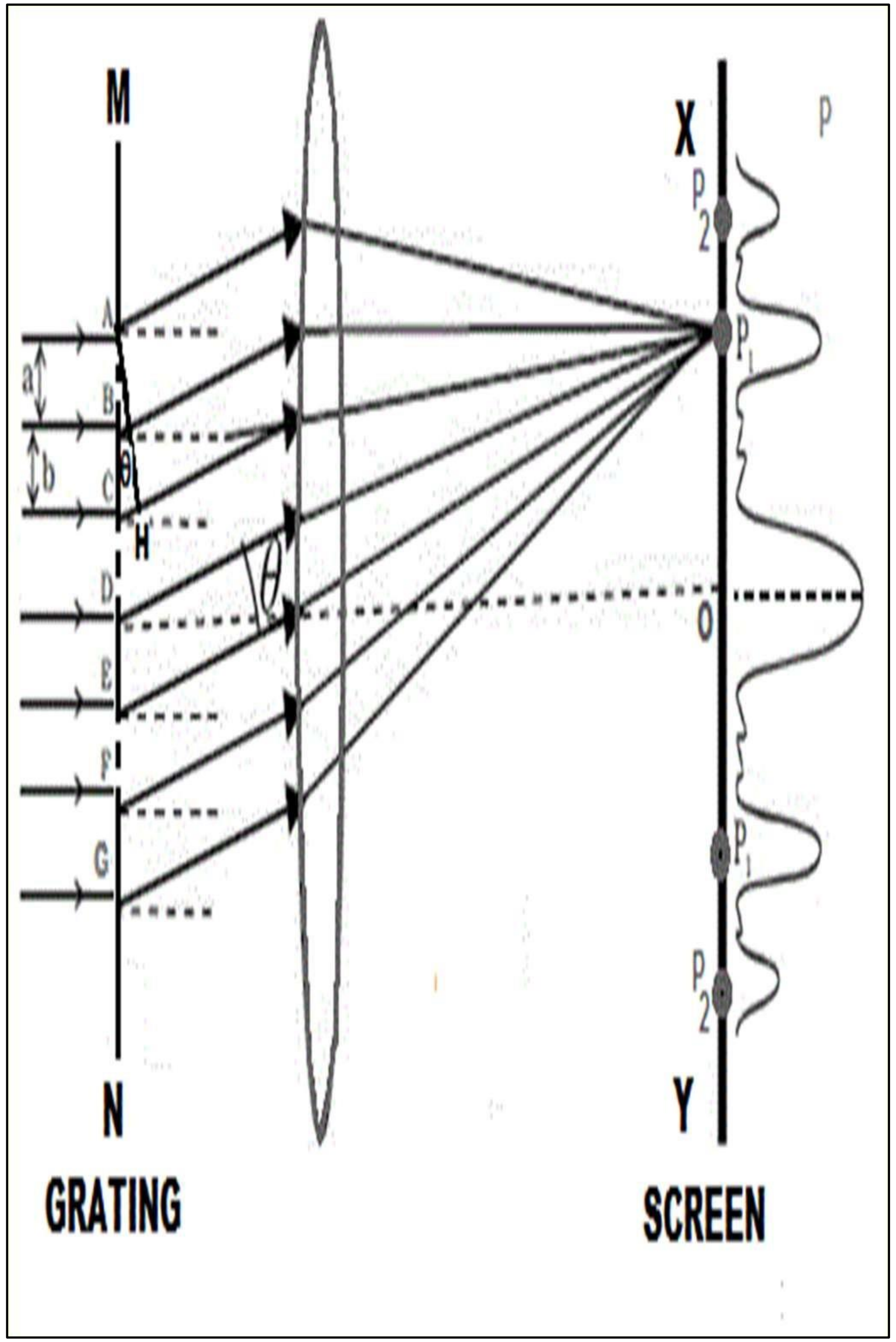
$$d \sin \theta = n\lambda$$

where 'd' is the distance between two consecutive slits and n =1, 2, 3----, is the order of principal maxima.

- This Equation is called Grating equation and gives the position of principal maxima.
- The rulings on the grating act as obstacles having a definite width 'b' and the transparent space between the rulings act as slit of width 'a'.
- The combined width of a ruling and a slit is called grating element d = a + b.



DIFFRACTION THROUGH PLANE TRANSMISSION GRATING



- Let MN is the plane transmission grating having AB, CD, EF as successive slits of equal width 'a' and BC, DE represent rulings of width 'b'.

- The path difference between the wavelets from one pair of corresponding points A and C is CH = (a + b) sin θ.

- The point P_1 will be bright, when

  $(a + b) \sin \theta = n \lambda$ where n = 0, 1, 2 3 --------

- Therefore $\sin\theta = \frac{n\lambda}{a+b}$

  or $\sin\theta = Nn\lambda$ where $N = \frac{1}{a+b}$, gives number of lines per unit width of the grating.

  Also from above equation $\therefore n = \frac{(a+b)\sin\theta}{\lambda}$

- Since the maximum angle of diffraction is 90°, hence the maximum possible order available in grating is given by

  $$\therefore n_{max.} = \frac{(a+b)}{\lambda} \text{ for } \sin\theta = 1$$

RESOLVING POWER OF GRATING

- ***Resolving power of the grating is defined as** the ability of a grating to form separate diffraction maxima of two wavelengths which are very close to each other.*

- **For two nearly equal wavelengths λ_1 and λ_2, between which a diffraction grating can just barely be distinguished, the resolving power 'R' of the grating is defined as**

- $$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{d\lambda}$$

where $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ is the mean value of the two wavelengths λ_1 and λ_2 and the smallest difference $d\lambda = \lambda_2 - \lambda_1$

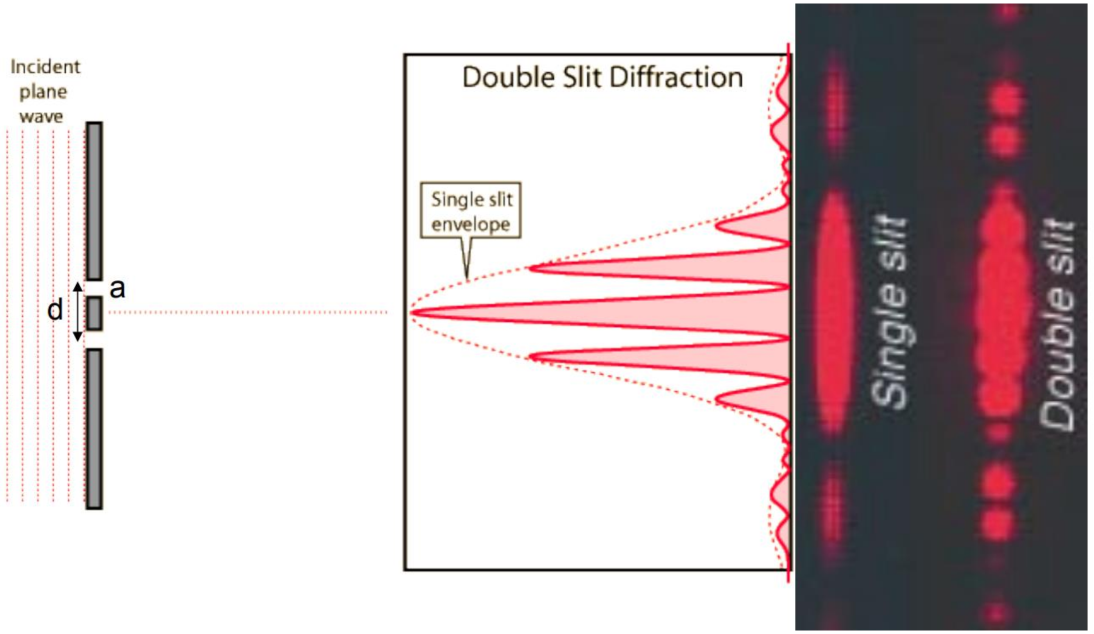
- **Also Resolving power of grating is found as**

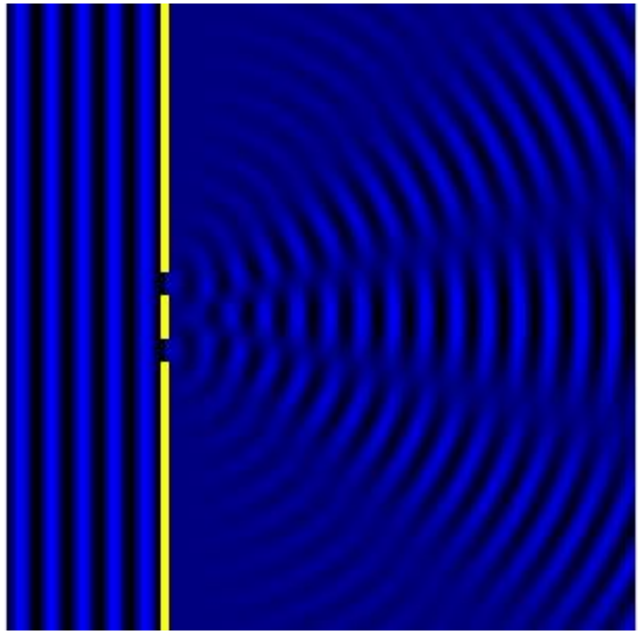
$$R.P. = \frac{\lambda}{d\lambda} = nN$$

Where the order of spectrum is 'n' and total number of lines on the grating surface 'N'.

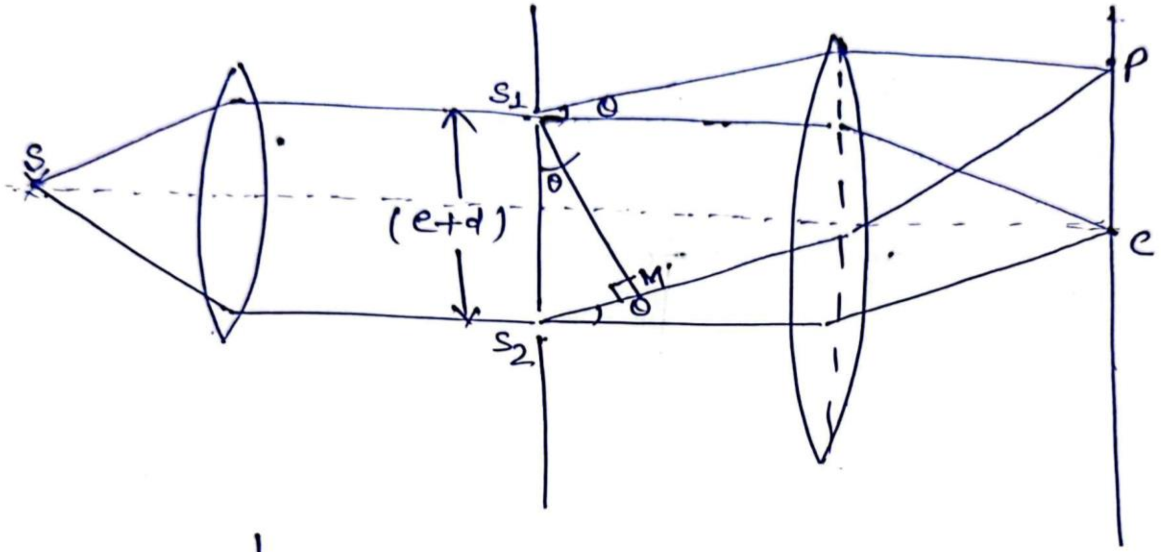
Fraunhofer diffraction due to Double-Slit

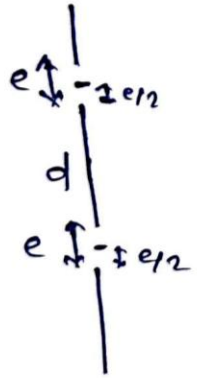
- When we studied interference in Young's double-slit experiment, we ignored the diffraction effect in each slit.
- We assumed that the slits were so narrow that on the screen, you saw only the interference of light from just two-point sources.
- If the slit is smaller than the wavelength, the diffraction phenomenon will occur.
- The diffraction pattern of two slits of width D that are separated by a distance d is the interference pattern of two point sources separated by d multiplied by the diffraction pattern of a slit of width D.





Fraunhofer's Diffraction Due to Double Slit



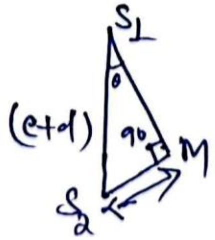


$S_1S_2 = e/2 + e/2 + d$

$S_1S_2 = (e+d)$

Path difference $S_2M = ?$

In triangle S_1S_2M



In ΔS_1S_2M, $\sin\theta = \dfrac{S_2M}{S_1S_2}$

or

$\sin\theta = \dfrac{S_2M}{(e+d)}$

$\therefore \quad S_2M = (e+d)\sin\theta$ ---- (1)

As we Know, Phase diff $= \dfrac{2\pi}{\lambda}$. Path diff

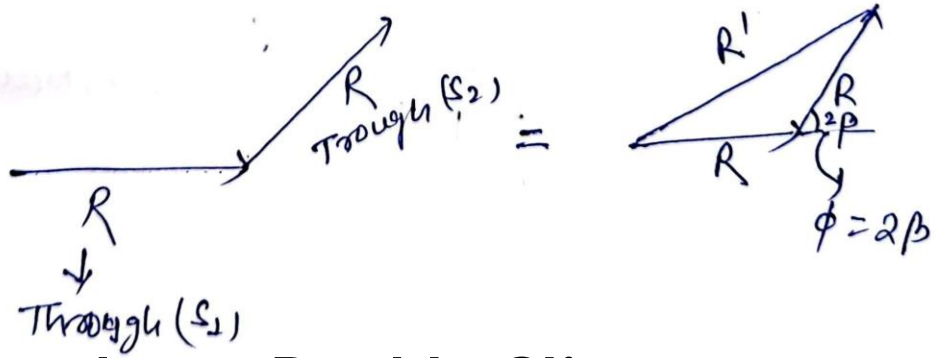
$\Delta\phi = \dfrac{2\pi}{\lambda}\cdot\Delta x$

$\therefore \Delta\phi = \dfrac{2\pi}{\lambda}\cdot(e+d)\sin\theta$ ---- (2)

Let us consider, $\Delta\phi = 2\beta$ ------ (3).

We have two slits, so we will get, diffraction pattern through both the slits.

therefore, from phasor dig diagram,



Fraunhofer diffraction due to Double-Slit

$$R'^2 = R^2 + R^2 + 2RR\cos 2\beta$$

$$R'^2 = 2R^2(1+\cos 2\beta)$$

$$= 2R^2(1+2\cos^2\beta - 1)$$

$$\boxed{R'^2 = 4R^2\cos^2\beta} \quad \text{-----} (4)$$

from single slit derivation,

$$R^2 = \frac{A^2\sin^2\alpha}{\alpha^2} \quad \text{......} (5)$$

therefore from (4) & (5)

$$R'^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta$$

$$I = 4A^2\underbrace{\left(\frac{\sin^2\alpha}{\alpha^2}\right)}_{\text{diffraction}}\underbrace{\cos^2\beta}_{\text{Interference}}$$

$$2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta$$

or

$$\boxed{\beta = \frac{\pi}{\lambda}(e+d)\sin\theta}$$

From single slit experiment, $I = R^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$

$\alpha = \frac{\pi}{\lambda}e\sin\theta$ (in the previous derivation we have show 'e' as 'd')

for Principle Maxima, $\theta = 0$

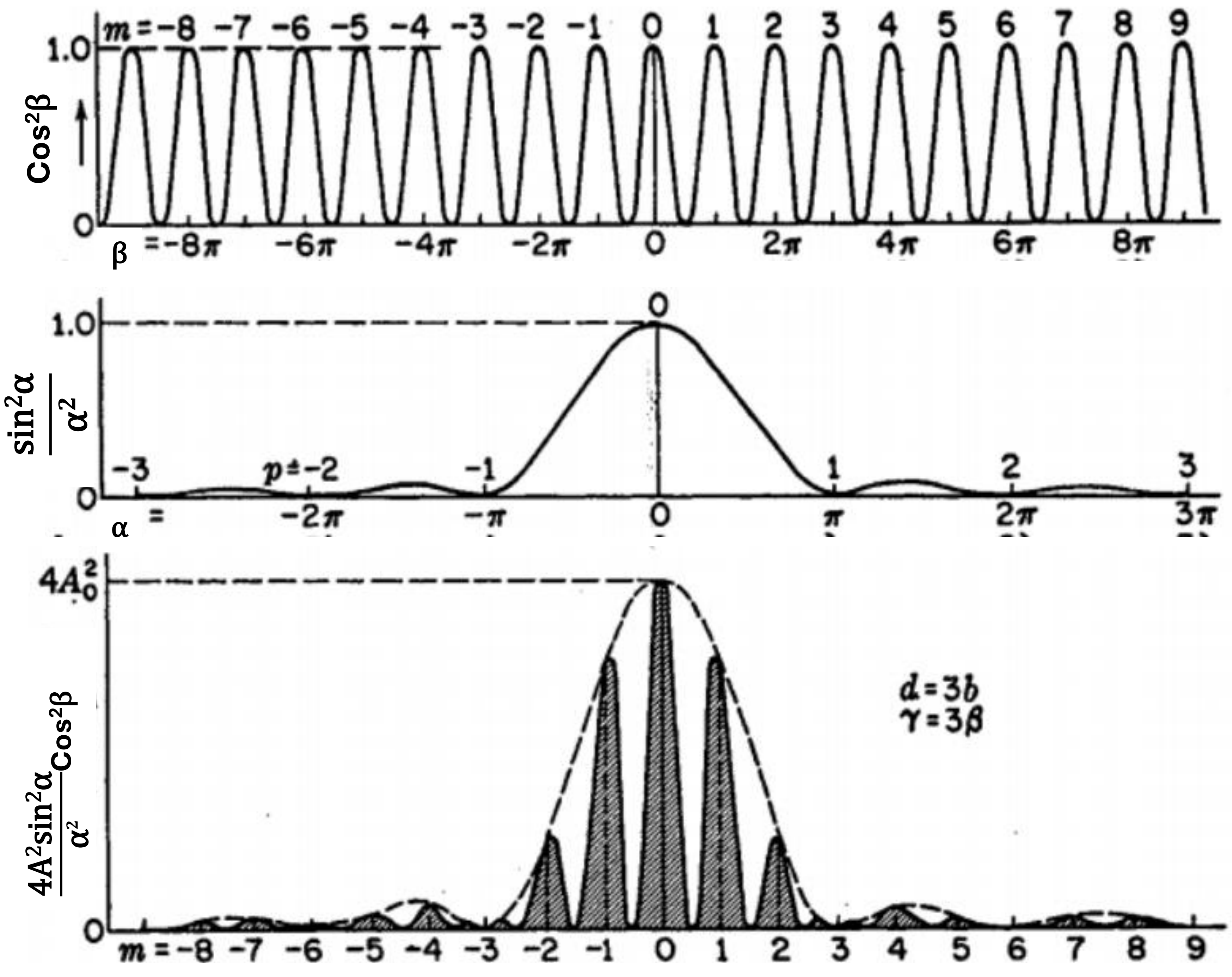
Secondary Minima, ~~$\alpha = \pm m\lambda$~~.

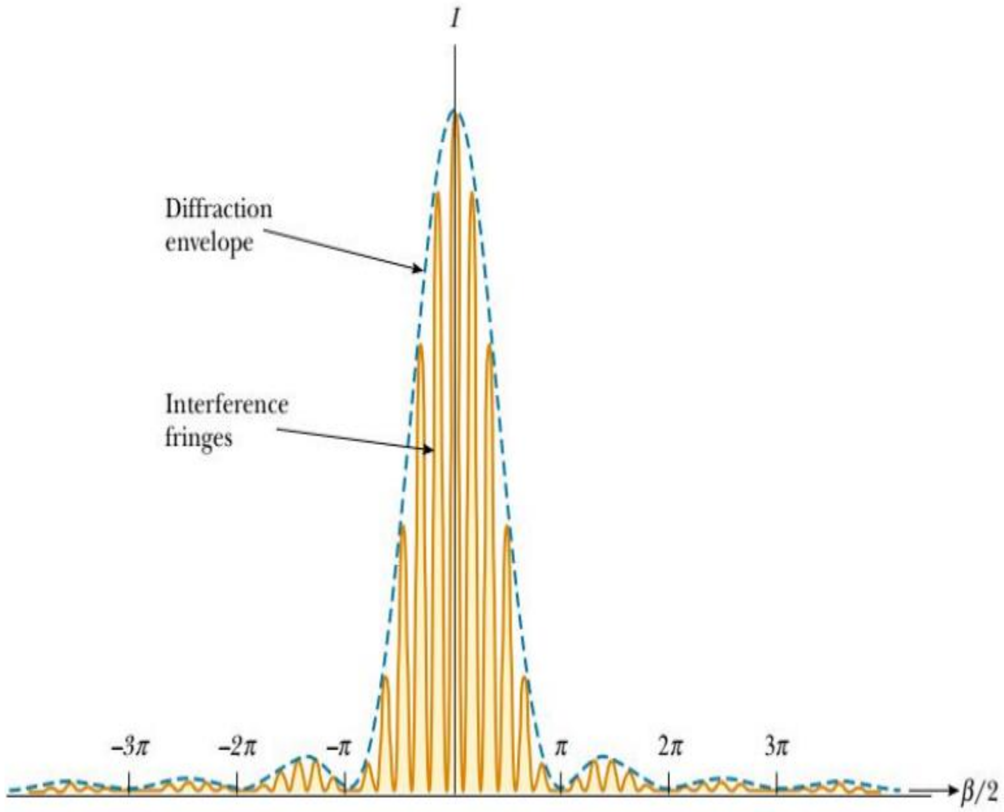
$$e\sin\theta = \pm m\lambda.$$

Secondary maxima. $\alpha = \pm\frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

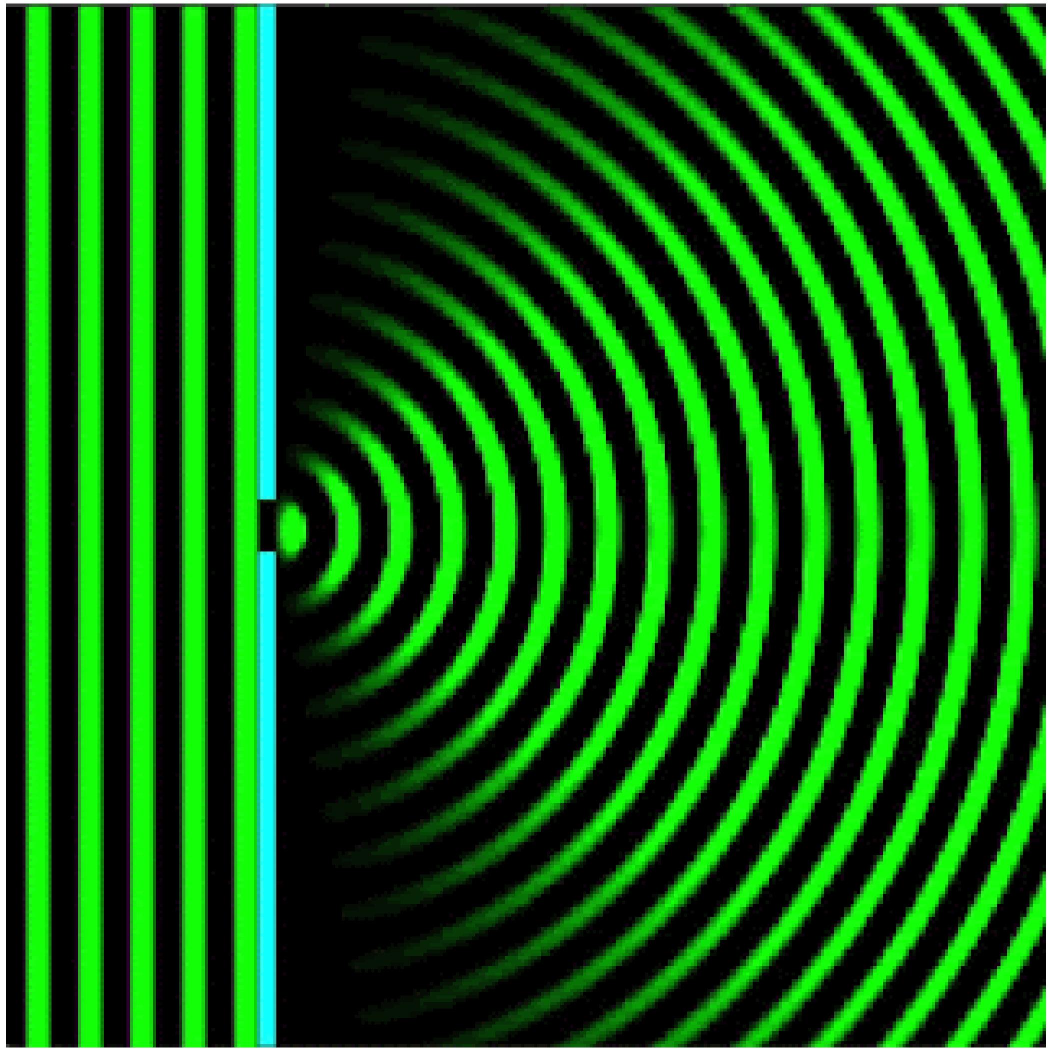
Fraunhofer diffraction due to Double-Slit

Fraunhofer diffraction due to Double-Slit

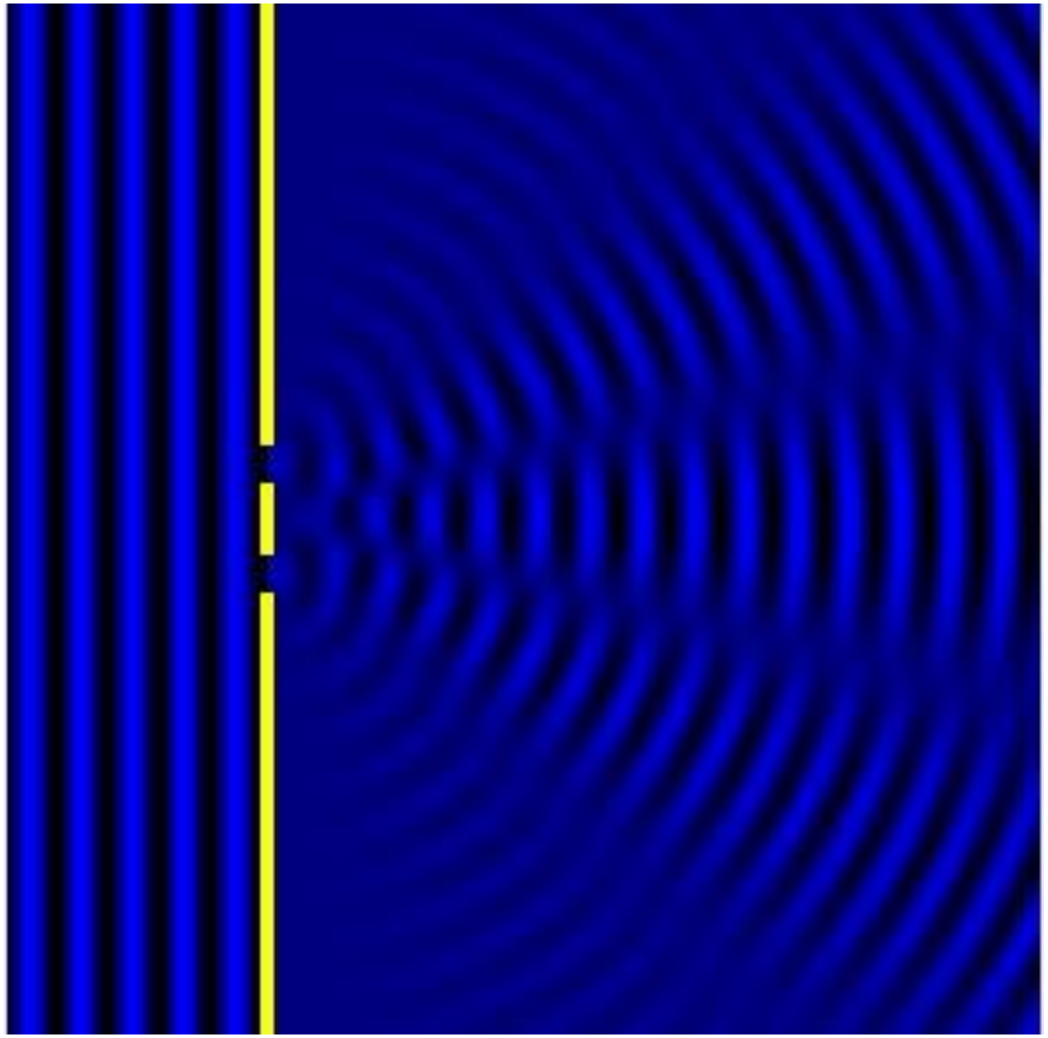




Fraunhofer diffraction due to single -Slit

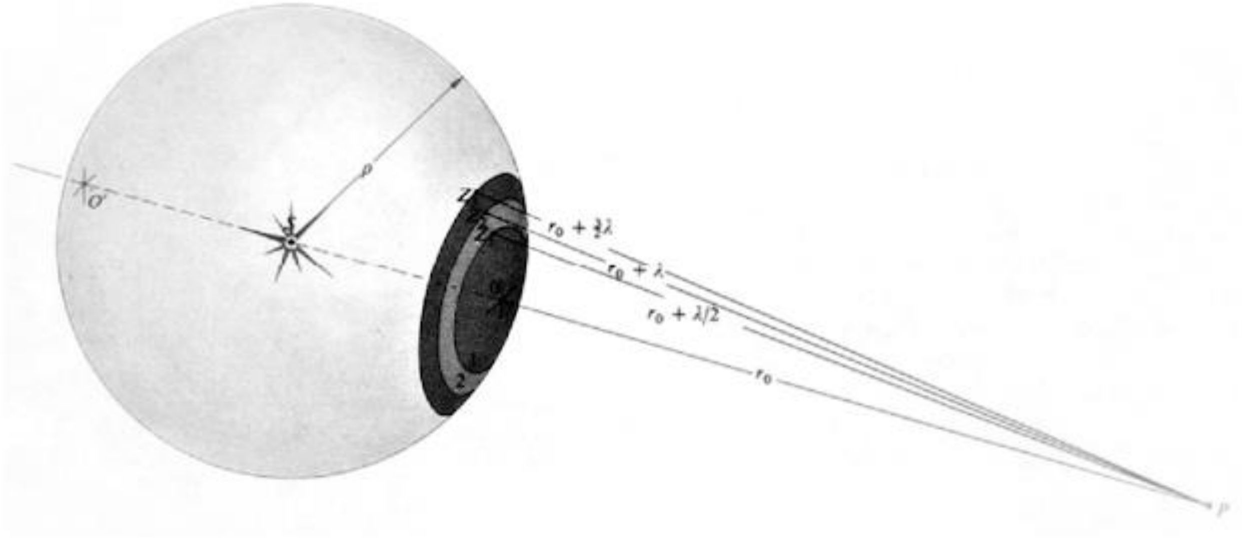


Fraunhofer diffraction due to Double-Slit

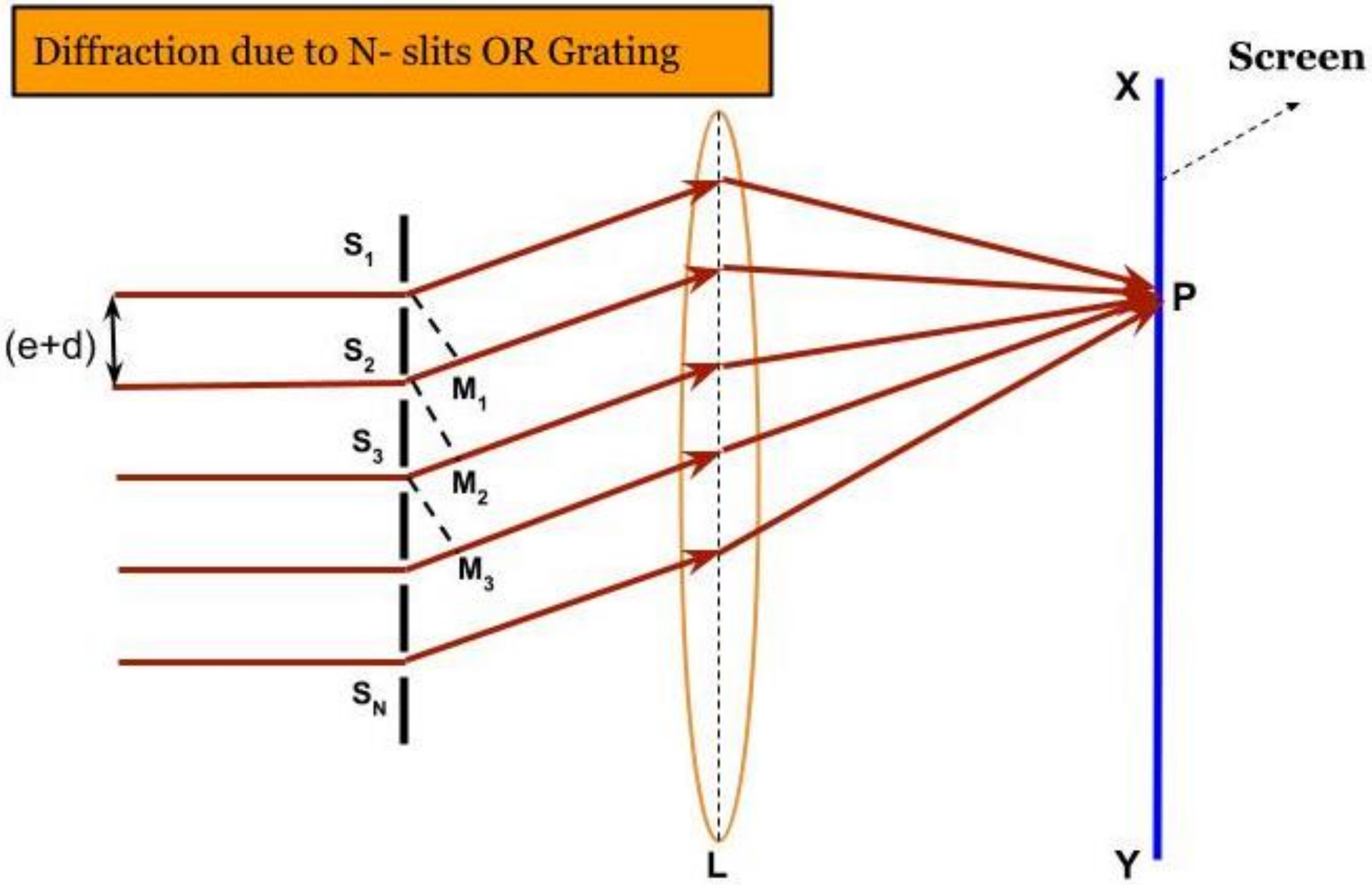


half-period zones for plane wavefront

half-period zones are essential components of **zone plates** used to focus light or form images by diffraction, similar to lenses. They are derived from **Fresnel diffraction theory** and are based on dividing the wavefront coming from a point source into concentric, alternating transparent and opaque zones.



Diffraction due to a plane diffraction grating or N- Parallel slits



Let e be the width of each slipped and d be the separation between any two consecutive slits then $(e + d)$ is known as the grating element. The diffracted ray from each slit, then $(e + d)$ is knowns as the grating element. The diffracted ray from each slit is focussed at a point P on the screen XY with the help of a convex lens L.

Let $S_1, S_2, S_3, \ldots\ldots$ be the middle point of each slit and $S_1M_1, S_2M_2, S_3M_3, \ldots\ldots\ldots S_{N-1}M_{N-1}$ be the perpendicular drawn as shown in the figure.The waves diffracted from each slit are equivalent to a single wave amplitude:

$$R = \frac{A\ sin\alpha}{\alpha} \qquad (1)$$

The path difference between the waves from slit S_1 and S_2 is

$$S_2M_1 = (e + d)sin\theta$$

The path difference between the waves from slit S_2 and S_3 is

$$S_3M_2 = (e + d)sin\theta$$

The path difference between the waves from slit S_{n-1} and S_n is

$$S_NM_{N-1} = (e + d)sin\theta$$

Thus, it is obvious that the path difference between all the consecutive waves is the same and equal to $(e + d)sin\theta$

The corresponding phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}(e+d)sin\theta \qquad (2)$$

Let $\Delta\phi = 2\beta$

$$\beta = \frac{\pi}{\lambda}(e+d)sin\theta \qquad (3)$$

Thus, the resultant amplitude at P is the resultant amplitude of N waves, each of amplitude R and its common phase difference is (2β)

$$R' = \frac{R\ sin\left(\frac{2N\beta}{2}\right)}{sin\left(\frac{2\beta}{2}\right)} \qquad (4)$$

The resultant amplitude at P

$$R' = \frac{R\ sinN\beta}{sin\beta}$$

Where $R = \frac{A\ sin\alpha}{\alpha}$. Now substitute the value of R in the above equation and we get

$$R' = \frac{A\ sin\alpha}{\alpha}\frac{sinN\beta}{sin\beta} \qquad (5)$$

The resultant intensity at P

$$I = R'^2$$

$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{sin^2N\beta}{sin^2\beta} \qquad (6)$$

The factor $\frac{A^2\ sin^2\alpha}{\alpha^2}$ gives the intensity pattern due to diffraction from a single slit while the factor $\frac{sin^2N\beta}{sin^2\beta}$ gives the distribution of intensity due to interference from all the N-slit

Principle Maxima→

The intensity will be maximum when $sin\beta = 0$ or $\beta = \pm n\pi$

Where $n = 0, 1, 2, 3, \ldots\ldots$

But under this condition, $sinN\beta$ is also equal to zero. Hence term $\frac{sinN\beta}{sin\beta}$ can be solve by

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(sinN\beta)}{\frac{d}{d\beta}(sin\beta)}$$

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta} = \lim_{\beta\to\pm n\pi}\frac{NcosN\beta}{cos\beta}$$

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta}=N\lim_{\beta\to\pm n\pi}\frac{cosN\beta}{cos\beta}$$

Where the value of $\lim_{\beta\to\pm n\pi}\frac{cosN\beta}{cos\beta}=1$

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta}=N \qquad (7)$$

So the maximum intensity

$$I=\frac{A^2\,sin^2\alpha}{\alpha^2}N^2 \qquad (8)$$

Thus, the condition for principle maxima

$$sin\beta=0$$

$$\beta=\pm n\pi$$

$$(e+d)sin\theta=\pm n\lambda \qquad (9)$$

For $n=0$, we get $\theta=0$ This $\theta=0$ gives the direction of zero-order principal maxima. For the value of $n=1,2,3,\ldots\ldots$, gives the direction of first, second, third,....... order principal maxima.

Minima →

The intensity will be minimum, when $sinN\beta=0$ but $sin\beta=0$

$$N\beta=\pm m\pi$$

$$N(e+d)sin\theta=\pm m\lambda \qquad (10)$$

Where m can take all integral values except $0,N,2N,3N,\ldots\ldots$ because for these values of m, $sin\beta=0$ which gives the position of principal maxima.

Secondary maxima→

It is obvious from the above condition of minima, there are $(N-1)$ minima between two successive principal maxima. Hence, there are $(N-2)$ other maxima with alternative minima between two successive principal maxima. These $(N-2)$ maxima are called secondary maxima. To find the condition of secondary maxima equation (6) is differentiated with respect to β and equated to zero.

$$\frac{dI}{d\beta}=\frac{A^2\,sin^2\alpha}{\alpha^2}2\frac{sinN\beta}{sin\beta}\left[\frac{sin\beta.N.cosN\beta-sinN\beta.cos\beta}{sin^2\beta}\right]$$

$$0=\frac{A^2\,sin^2\alpha}{\alpha^2}2\frac{sinN\beta}{sin\beta}\left[\frac{sin\beta.N.cosN\beta-sinN\beta.cos\beta}{sin^2\beta}\right]$$

$$N.sin\beta.cosN\beta-sinN\beta.cos\beta=0$$

$$\boxed{\tan N\beta=Ntan\beta \qquad (11)}$$

$$sinN\beta = \frac{Ntan\beta}{\sqrt{1+N^2tan^2\beta}} \qquad (12)$$

Substituting the value of $sinN\beta$ from the above equation to equation (6)

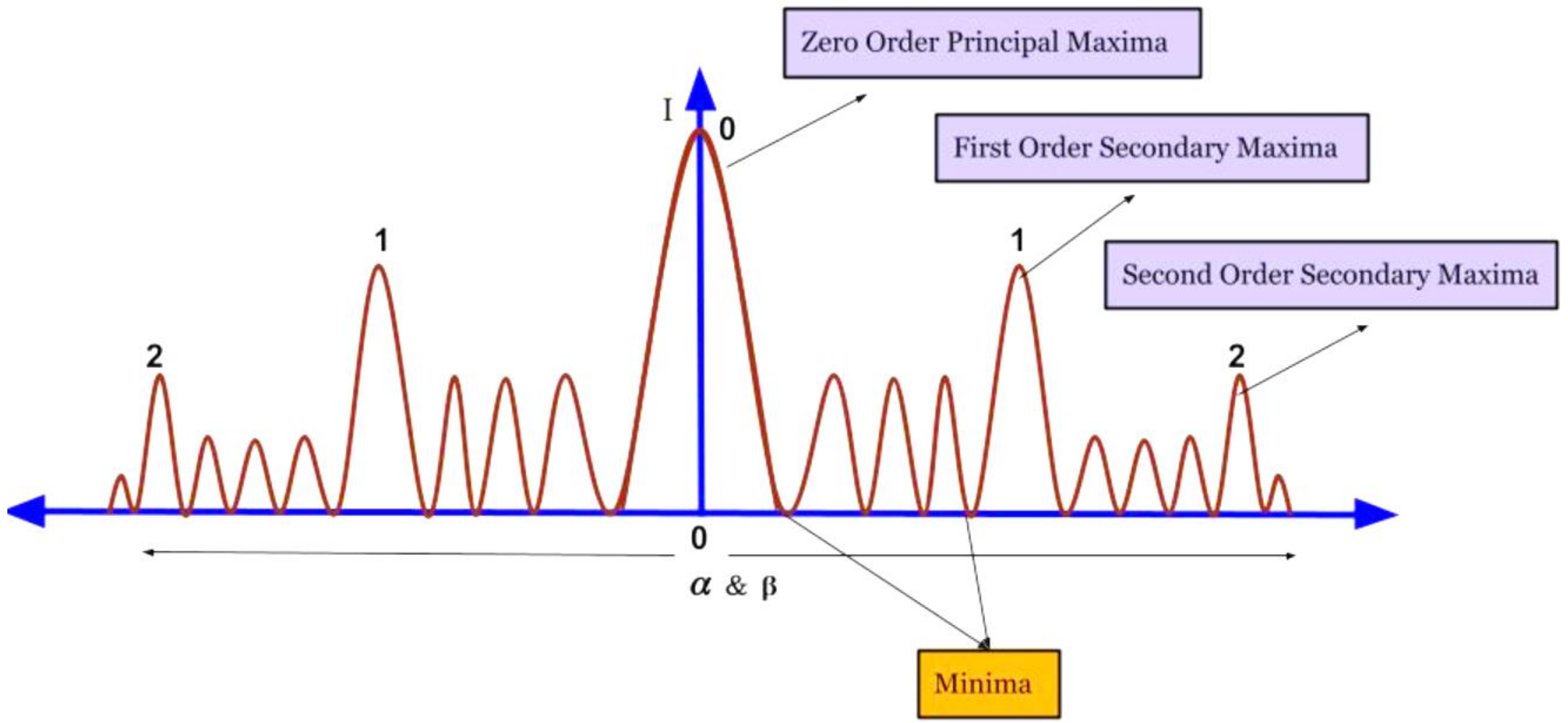
$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{N^2tan^2\beta}{1+N^2tan^2\beta}\frac{1}{sin^2\beta}$$

$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{N^2}{1+N^2tan^2\beta}\frac{1}{cos^2\beta} \qquad \left(\because tan\beta = \frac{sin\beta}{cos\beta}\right)$$

$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{N^2}{cos^2\beta+N^2sin^2\beta}$$

$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{N^2}{1-sin^2\beta+N^2sin^2\beta}$$

$$I = \frac{A^2\ sin^2\alpha}{\alpha^2}\frac{N^2}{1+(N^2-1)sin^2\beta} \qquad (12)$$

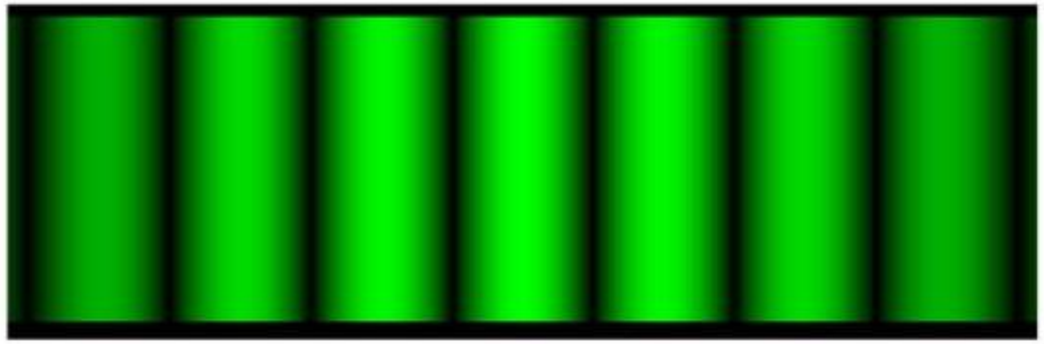


Now divide the equation (12) by equation (8) so

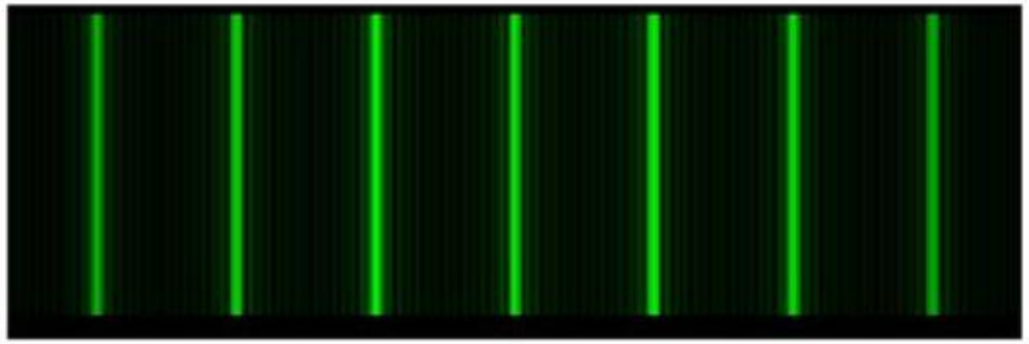
$$\frac{Intensity\ of\ secondary\ maxima}{Intensity\ of\ principal\ maxima} = \frac{1}{1+(N^2-1)sin^2\beta}$$

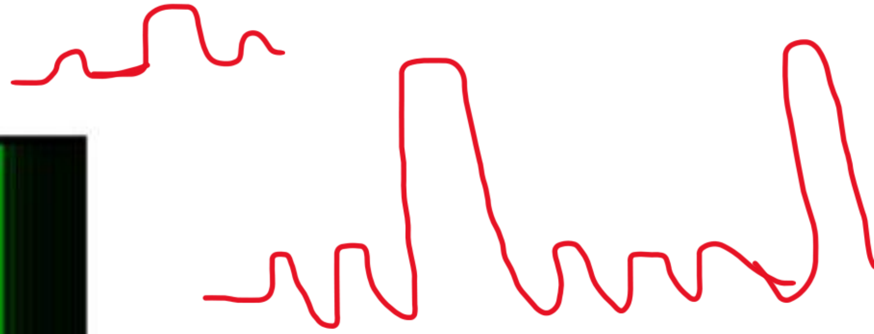
It is obvious from the above equation that When N increases then the intensity of secondary maxima decreases.

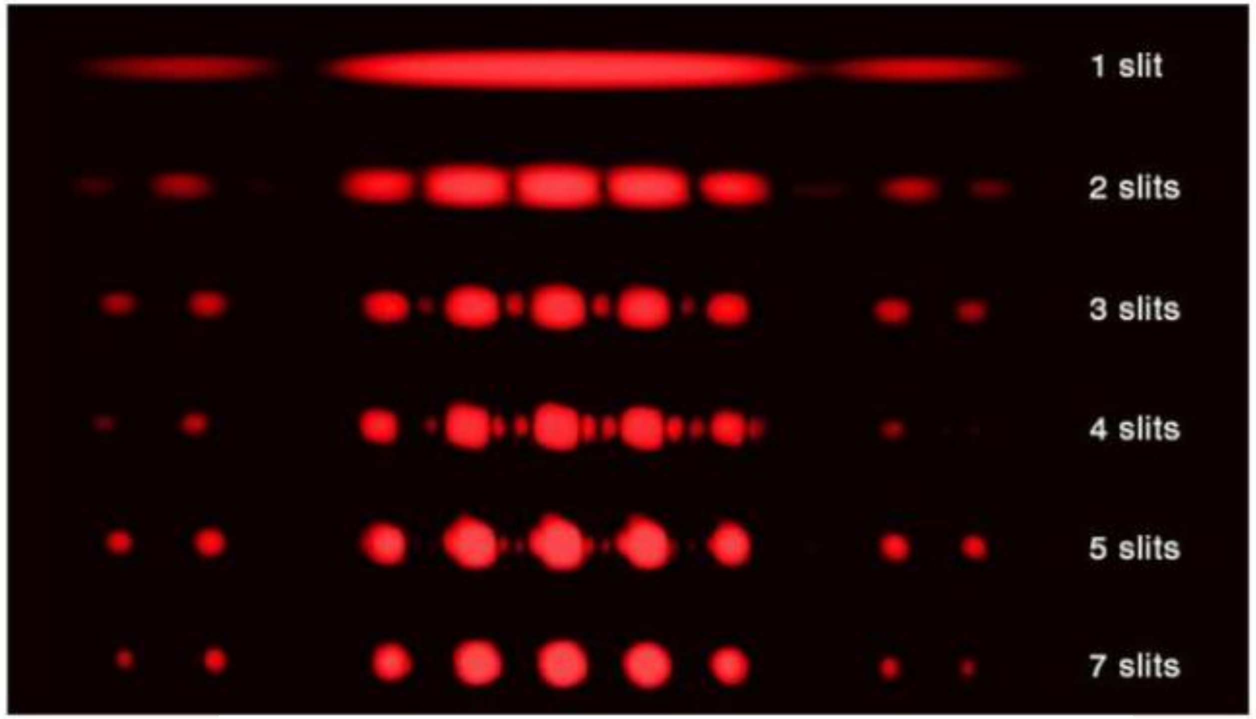
Double Slit



Diffraction Grating

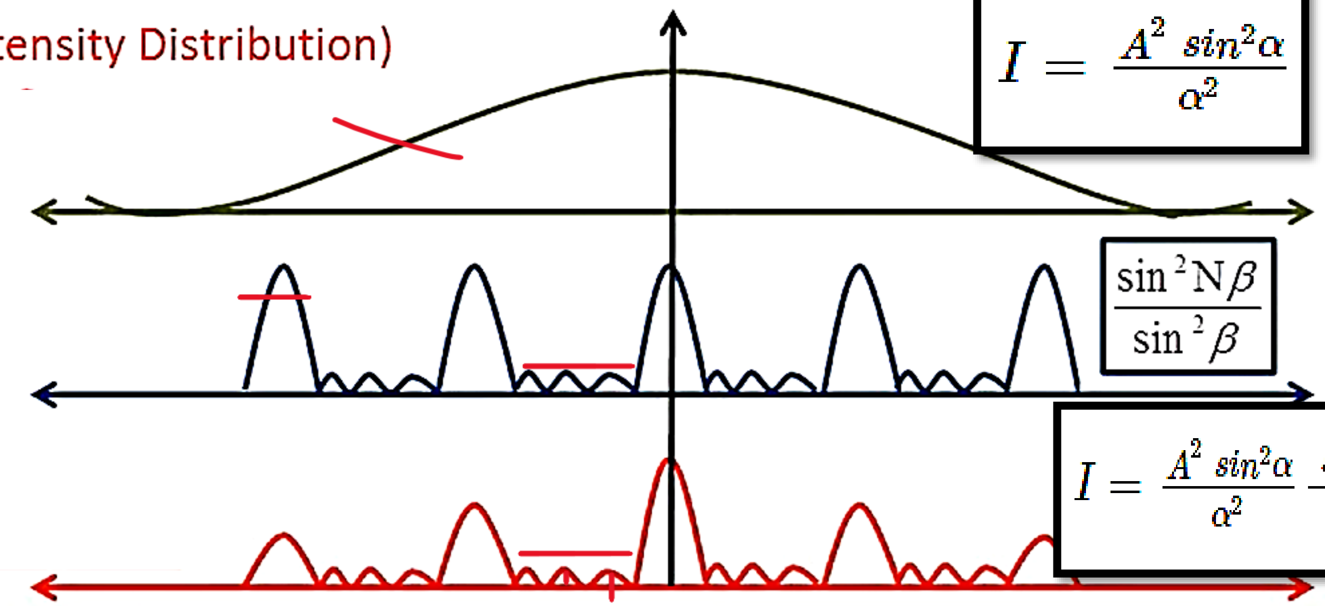






Fraunhofer Diffraction at N parallel Slits

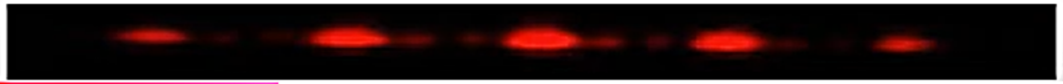
(Intensity Distribution)



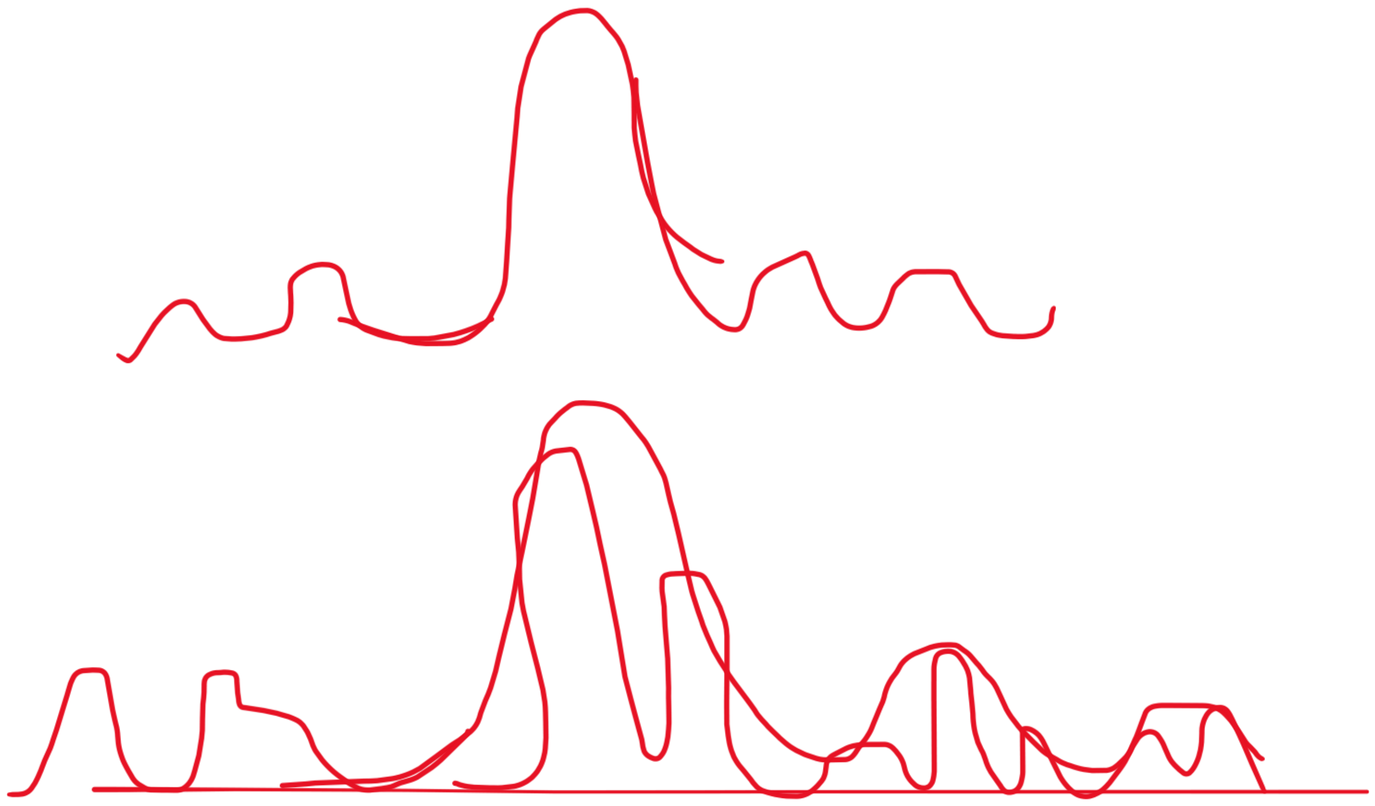
$$I = \frac{A^2 \sin^2\alpha}{\alpha^2}$$

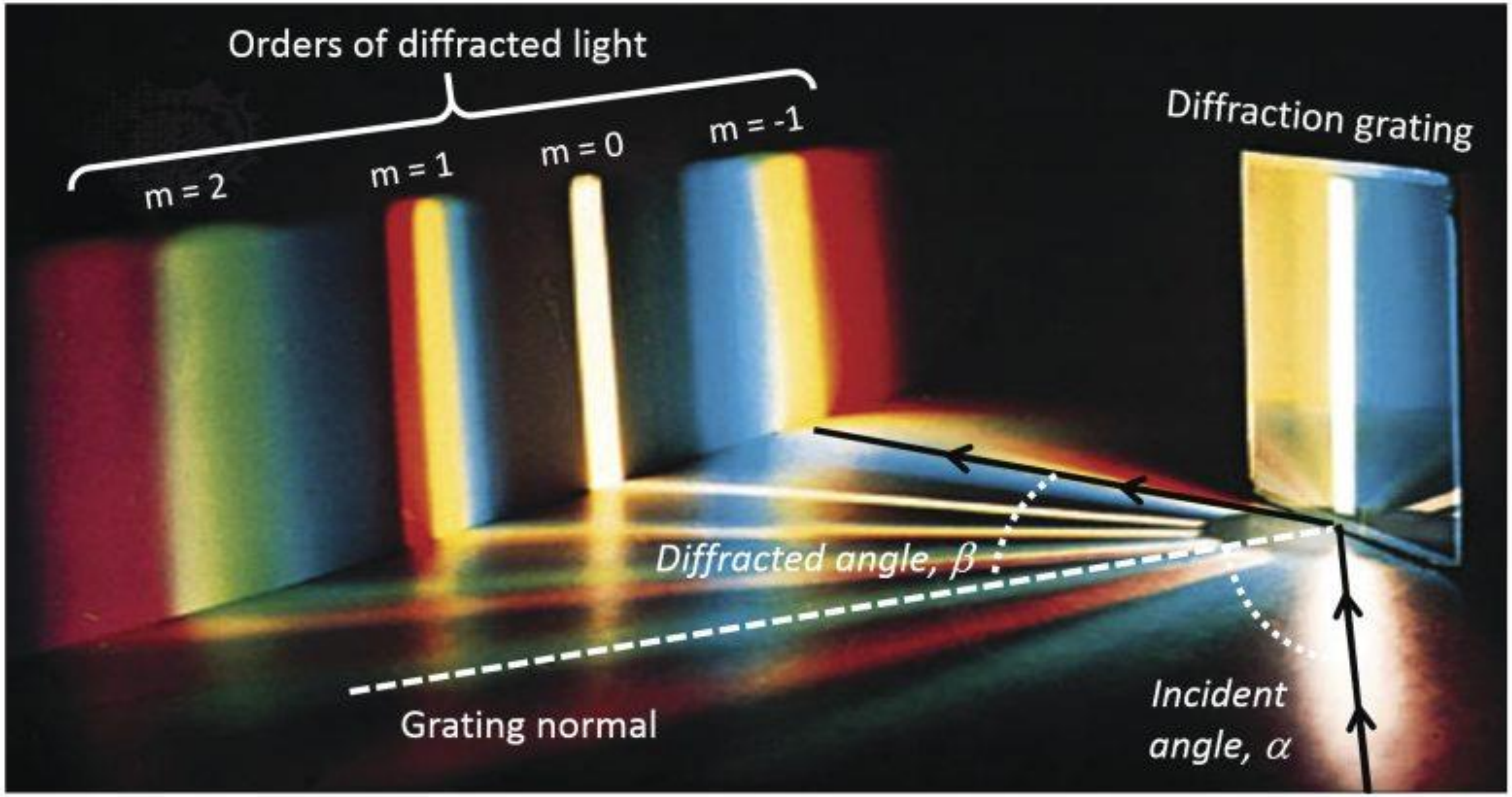
$$\frac{\sin^2 N\beta}{\sin^2\beta}$$

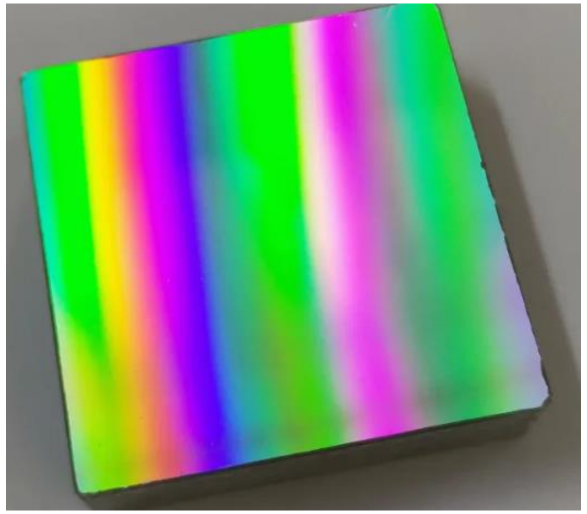
$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$



$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

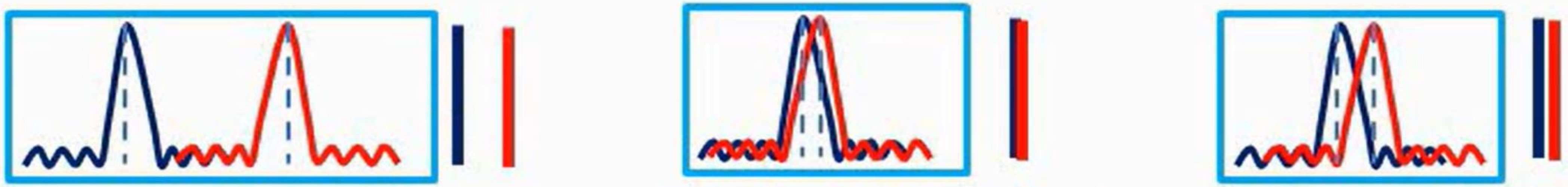



Orders of diffracted light
m = 2
m = 1
m = 0
m = -1
Diffraction grating
Diffracted angle, β
Grating normal
Incident angle, α



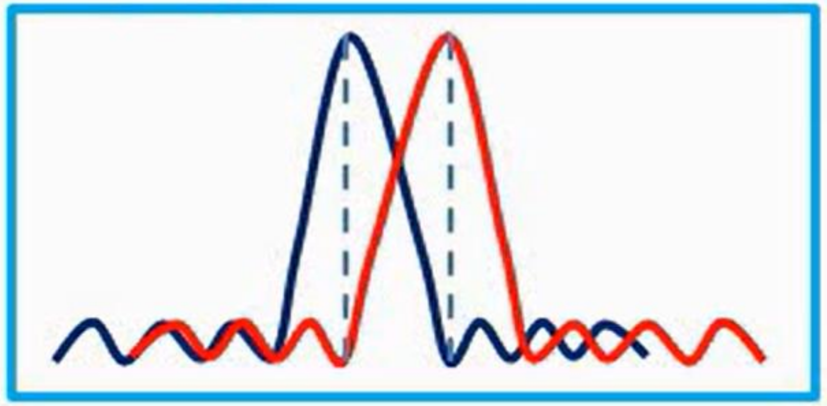
Resolving Power of a Grating

Resolving power of grating is defined as its ability to produce separate principal maxima for two wavelengths λ_1 and λ_2



It states that two wavelengths are just resolved if principal maximum of one coincides with the first minimum of the other and vice versa

Rayleigh's criterion of just resolution



$$\text{R.P.} = \frac{\lambda}{d\lambda}$$

The resolving power of a grating is its ability to distinguish between two closely spaced wavelengths in the spectrum. It is mathematically expressed as:

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

where:

- R is the resolving power,
- λ is the wavelength of light being analyzed,
- $\Delta\lambda$ is the smallest difference in wavelengths that can be resolved,
- m is the diffraction order,
- N is the total number of illuminated slits (grating lines) in the grating.

Mathematical Derivation

1. **Condition for diffraction maxima:** A diffraction grating produces maxima at angles wl path difference between adjacent slits is an integral multiple of the wavelength, given

$$d \sin\theta = m\lambda$$

- d is the grating spacing (distance between adjacent lines),
- θ is the diffraction angle for the m^{th} order.

2. **Resolving power definition:** Two wavelengths λ and $\lambda + \Delta\lambda$ are resolved when their diffraction maxima are distinguishable. This corresponds to the condition that the angular separation of their maxima is at least the angular width of a single diffraction peak.
3. **Width of a diffraction peak:** For N slits, the angular width of a single peak is inversely proportional to N, since the total spread of the interference pattern narrows with increasing N. Hence, the grating provides sharper peaks for larger N.
4. **Resolving power relationship:** For the m^{th} order diffraction, the resolving power is derived from the condition:

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

This shows that the resolving power increases with:

- Higher diffraction orders (m),
- Larger numbers of slits (N) illuminated by the incident beam.

Example:

If a grating has $N = 5000$ lines illuminated and is used in the second-order ($m = 2$), the resolving power is:

$$R = mN = 2 \times 5000 = 10,000$$

This means it can distinguish two wavelengths with a separation of:

$$\Delta\lambda = \frac{\lambda}{R}$$

For $\lambda = 500\,\mathrm{nm}$, the smallest resolvable difference is:

$$\Delta\lambda = \frac{500\,\mathrm{nm}}{10,000} = 0.05\,\mathrm{nm}.$$

Fresnel and Fraunhofer diffraction. Fresnel's Half-Period Zones for Plane Wave. Fresnel diffraction pattern of a straight edge and at a circular aperture. Fraunhofer diffraction: Single slit. Double slit. Diffraction grating. Resolving power of grating.

Syllabus